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AN ANALYTICAL APPROACH TO INELASTIC BEHAVIOR OF REINFORCED CONCRETE WALLS SUBJECTED TO REVERSED CYCLIC LOADING

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ABSTRACT

A computer program called WCOMR was developed to analyze the inelastic behavior of reinforced concrete walls subjected to reversed cyclic loadings. The WCOMR comprises reinforced concrete elements described by a smeared crack model and joint elements described by a discrete crack model. The applicability of the WCOMR has been experimentally verified by the element and structure levels.

1. INTRODUCTION

In making analytical predictions for inelastic behavior of reinforced concrete walls subjected to reversed cyclic loadings, modeling of reinforced concrete elements, including the cracking habits, from yielding of the reinforcing bars and on through subsequent repetition of loading, is indispensable. As an reinforced concrete wall contains steel bars dispersedly disposed in two or more directions, numerous cracks are generated dispersedly. Moreover, behaviors of cracks after they have all been generated and attained a stable state are more important than the generation and development of individual cracks. Therefore, the smeared crack model, in which a finite region that contains several cracks and reinforcing bars are considered to be a continuum, is quite adequate to describe a reinforced concrete element.

However, reality is that local discontinuities, like slipping of bars and intrusion of junction planes, can take place due to abrupt changes induced at the joints of elements in any composite structural parts composed of several different basic structural elements of which the reinforced concrete wall is one as shown in Fig.1. To take these effects into account, the smeared crack model alone is inadequate, and introduction of the discrete crack model becomes necessary. In this paper, a computer program called WCOMR is presented for FEM analysis of reversed cyclic response of reinforced concrete walls by combining these two models.

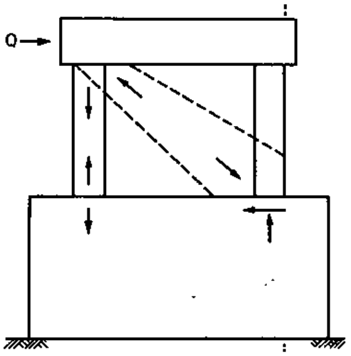

Fig.1 An example of reinforced concrete wall

2. DEVELOPMENT OF ANALYTICAL MODEL

2.1 Starting Points of the Analytical Model

There are many models for FEM analysis of reinforced concrete walls. For example, solutions for a reinforced concrete wall as a whole can be obtained by formulating empirical results of deformation and fracture of its elements under macroscopic stresses, and by using their characteristics as the elements in the finite elements. It is also possible to solve such a wall by analyzing the behaviors of individual elemnts, where each element, such as the steel bars, the concrete, cracks, and bond, are modeled independently. Althouth the method of approach may differ, any models have a set of empirical formulae, the only difference being where the modeling for analysis has begun (Okamura et al, 1986). It is quite natural that the closer the analytical model has its origin to the problems, the greater is its accuracy. It is important to note, however, that the closer to the level 1 in Fig.2 the model has been elected to originate, the wider the application it becomes able to claim. It is with these in mind that comprehensive analytical model has been developed that starts off at the level 1 of Fig.2 to describe accurately the individual kinetic behaviors, including those under reversed cyclic loadings.

2.2 Analytical Models for Reinforced Concrete Elements

The analytical models for reinforced concrete elements fall on the level 2 in Fig.2, meaning that they may be constructed by combining the level 1 models. Care should be taken for the fact that, judgment of their fitness as a reinforced concrete element must be done by comparison with experiments conducted on the reinforced concrete element level (Okamura et al, 1986), since conditions of experiments for constructing micro models are often limited and idealized. Monotonic loading experiments by Collins and Vecchio (1982) and by Aoyagi and Yamada (1984), and reversed cyclic loading experiments by Stevens et al. (1987) are worth referring to. Their results were first verified as capable of simulating both the empirically determined envelopes and internal history curves quite well, then were used in proving that the level 1 model developed was applicable to level 2 (Izumo et al, 1989). However, one deficiency with this model is that, since its micromodels are empirical formulae, each describing faithfully an individual behavior (Maekawa and Okamura, 1983)(Li et al, 1989)(Shims et al, 1987), the model as a whole becomes too complex, and it is difficult to apply it straight to the FEM analysis of level 3 in a form including the path dependency under cyclic loads. As this called for modifications to make FEM analysis possible, a series of level 2 analyses was conducted to develop a reinforced concrete model applicable to level 3. The outcome has been a comparatively simple model, and this report presents the results of analysis conducted for reinforced concrete walls based on this model.

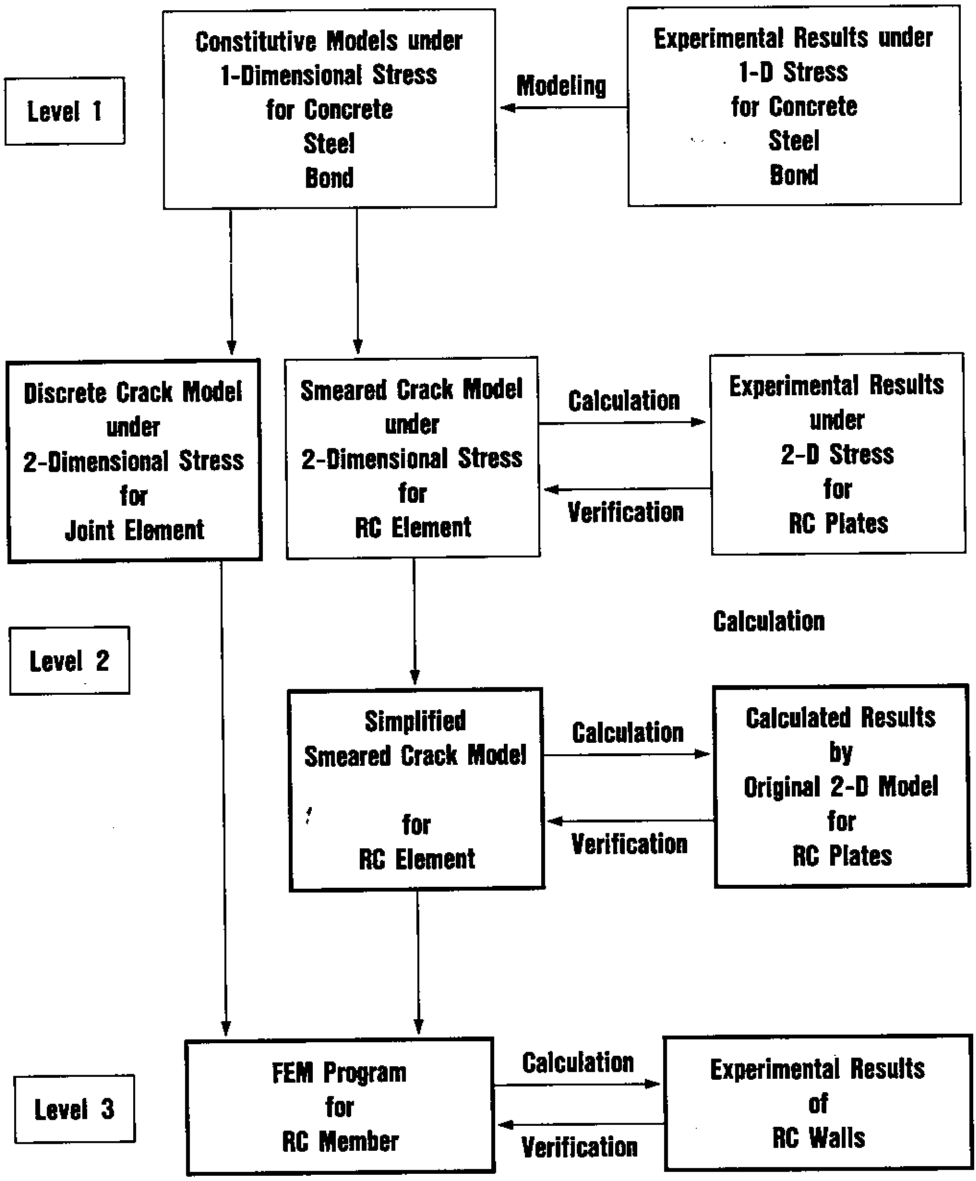

Fig.2 Development and verification of FE analysis for reinforced concrete

2.3 Analytical Model for Reinforced Concrete Walls

As mentioned briefly, the kind of reinforced concrete walls that are the object of this work are generally composed of such basic structural elements as wall, beam, and column, justifying the FEM analysis to be conducted for reinforced concrete walls by combining these reninforced concrete elements. It is to be remembered here that local discontinuities, such as pull-out of steel bars and slipping or intrusion of junction plane, can and do often take place as a result of abrupt changes in the section stiffness occurring at the joint planes connecting two components of different thicknesses, and that many FEM analyses have given load-displacement relations that run higher to the observed ones because their influences are overlooked.

On the other hand, however, no attempts should be made to alter the level 2 of reinforced concrete element models so as to accommodate them to the observed load displacement relationships in conducting critical comparison with experimental results in this level. This is because FEM analysis that has taken the effects of local displacements comprehensively into the reinforced concrete element model may become inapplicable to different walls in scale from the test specimens used for critical comparison, even while it is quite capable of applying itself to specimens if the scales are the same, since the proportions of the local displacements to the overall displacement differ by the size and the form of the specimens. To overcome this difficulty, it is necessary to provide a sophisticated joint element that describes the stress versus localized deformation relationship of the junction plane between two reinforced concrete elements of different sections. This has been done by devising a joint model on the basis of the discrete crack model to be employed to connect two planar elements of different sections.

The analytical model was constructed by going back to the level 2 in Fig.2, and by modifying each one of the level 1 micro models for structural elements, such as the bond slip of bars and shear slip of the junction plane according to the results of numerical experimenmts. As will be discussed later, one significance of clarifying the influences of local displacements at the junction planes in FEM analysis of reversed cyclic response of reinforced concrete walls lies in the fact that analytical predictions become possible for the reversed cyclic response of real size structures, the properties that are difficult to examine with reduced size test specimens because of the size effect.

3. DEVELOPMENT OF THE COMPUTER PROGRAM

The new FEM program was developed by incorporating the reinforced concrete element model and the joint element model described above into the COMM2 (Maekawa et al, 1983) that was formulated previously.

3.1 Inelastic Solutions

Since the present analysis is for reversed cyclic loadings, the numerical analysis is as complex as the analytical model compared with the case of monotonic loadings. In this section, the method of obtaining inelastic solutions is described. In the first place, the stress on a reinforced concrete element is obtained, not by the conventional method of step-by-step integration of the tangential stiffness, but by directly computing stresses from the total strain given by the constitutive relations as shown in Fig.3. Therefore, the stiffness matrix used in the iterative calculation is to correct the assumed strains for the unbalanced forces present. In order words, the matrix is determined so as to make the calculation convergence most effectively. The matrix and the element model used are as follows:

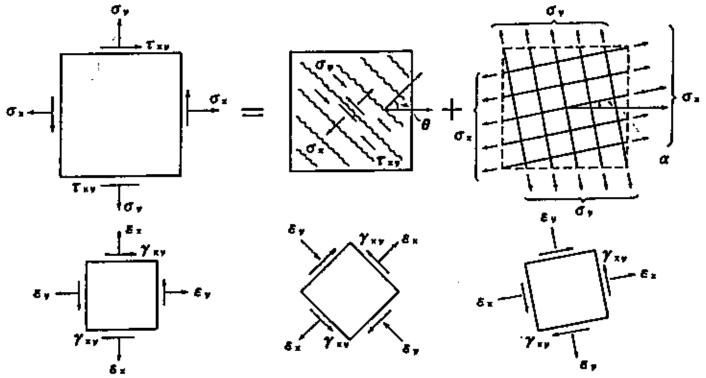

Fig.3 Reinforced concrete plate element model

(1) No negative stiffness for iteration scheme shall be used even when the tangential stiffness has become negative. This is to prevent the iterative solution from diverging as the stiffness matrix becomes singular provided that the material stiffness has assumed a negative value.
(2) The larger of the two stiffnesses shall be taken in the iterative calcualtion whenever the stress-strain relation changes. This is to suppress oscillation of solution in the convergent calculation.
(3) Judgment whether a finite element is under unloading or reloading shall be done with reference to the structural loading condition.
(4) The stress-strain relation of the element model shall conserve its continuity under any loading.
(5) In calculting stresses from strains in the element model, no implicit models that necessarily call for iterative operation to obtain stresses shall be used. This is to prevent the computation time from prolonging unduly because of the iterative convergence calcualtions that are conducted for each of the elements, and because of the difficulty of efficiently obtaining the stiffness that is inherent in such methods.
(6) Even though the stiffness matrix is revised for each iteration up to the fifth round, by which time such large changes in the stiffness as occurrence of cracks or of yielding of the steel bars should have occurred, no revision is to be made for further rounds like in the modified Newton-Raphson method.

3.2 Method of Dealing with Bi-Directional Cracking

Stresses working in a reinforced concrete element can be calculated from the incidental strains in the model described above by performing coordinate transformation, with the crack as the reference axis (local coordinate system) for the concrete, and with the x-axis of the steel bars as the reference axis for the reinforcement. The closing of cracks that takes place during cyclic loading, on the other hand, is taken into account by the tension stiffening model (Shima et al, 1987), which was developed for unloading and reloading considering bond of steel bars to concrete and of contact of the two surfaces of a crack.

When a second crack has been generated, however, the reference axis is transferred from the first crack to the second. In this case, deformation at the closed crack is ignored. The stiffness of a reinforced concrete element is determined by the deformation occurring at the surfaces of the reference crack whose cracking width is larger, provided that the angle the two cracks made is not smaller than 15 deg. Subsequent change of the reference crack in any loading cycle is conducted when, and only when, the stress working normal to that crack concerned has become smaller than that for the other crack.

4 FEM ANALYSIS OF REVERSED CYCLIC RESPONSE OF REINFORCED CONCRETE WALLS

4.1 General Consideration

One important consideration that has to be given to any FEM analysis is how to get correct results in the shortest possible computation time, and the finite element discretization and the load divisioning are the two major influencing factors.

The general trend is that the finer an element is divided, the more rigorous the solution becomes, although the longer becomes the computation time. In the case of conducting analysis involving the assumption of the smeared crack model as the present FEM analysis does, however, the solution does not necessarily become more rigorous, with finer elements, or more elements. The smeared crack model assumes a finite region that contains several cracks and reinforcing bars as a continuum, and then divide an element into a finer size than the interspacing of the generated cracks is to transcend the boundaries imposed. For analysis of this sort, therefore, methods that use elements of a high accuracy should be more appropriate than those that call for fine element divisioning.

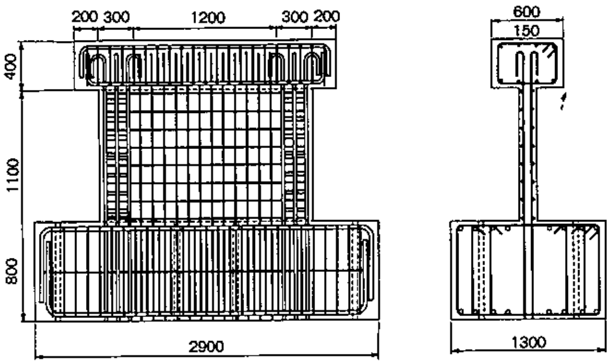

Fig.4 (a) Reinforced concrete wall specimen

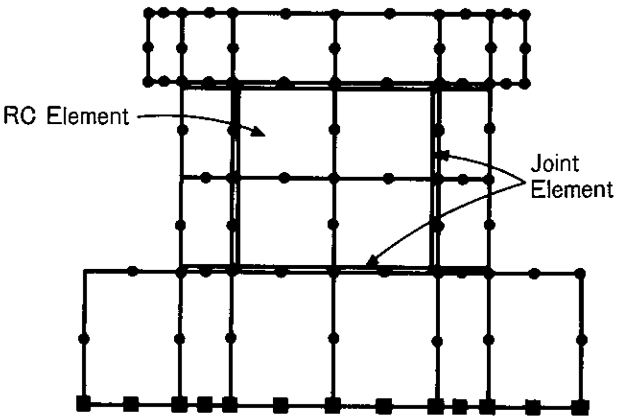

Fig.4(b) Finite element mesh

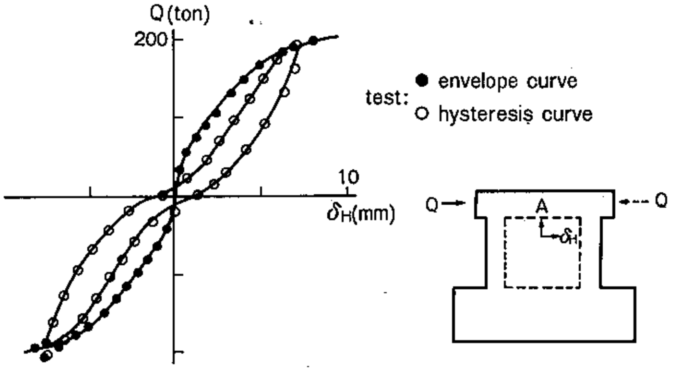

Fig.4(c) Horizontal displacement at point A

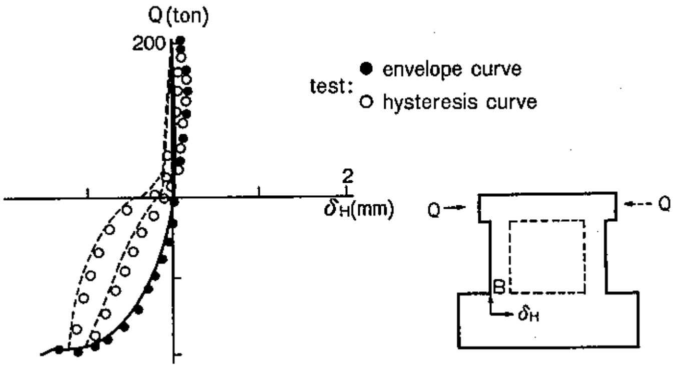

Fig.4(d) Horizontal displacement at point B

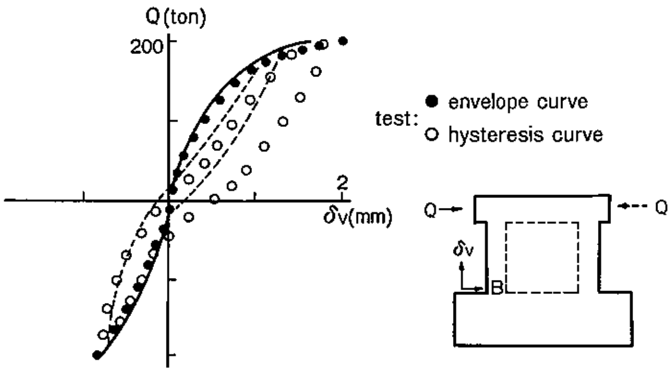

Fig.4(e) Vertical displacement at point B

For the problem at hand, comparison was made for the method of element dicretization against that of halving the element size doubling the number of elements, by analyzing the reinforced concrete wall of Fig.4(a). Since both were found to give much the same results, the element divisioning shown in Fig.4(b) was concluded to be quite adequate for obtaining practical solutions. The element used in the present work for the purpose of raising the accuracy is the 8-node quadrilateral isoparametric element.

In following the reversed cyclic response of a reinforced concrete wall from generation of cracks up to the ultimate load, or through the cycles of repeated unloading and reloading, the magnitude of each load increment or each forced displacement in the case of displacement controlled experiments experts a grave influence on the computation time. Even though large load increments or large forced displacements will reduce the number of steps needed for analysis, on the other hand, not only will the number of calculations to attain convergence for each step increase, but the calcualtion itself will not converge if the assigned load step or the forced displacement happens to be larger than a certain value.

By trial and error, it was found that the magnitude of a loading step or a forced displacement could be as much as 5 to 10 % of the ultimte load or the displacement at the ultimate load. Nevertheless, each forced displacement was held to less than 5 % of the ultimate displacement in this work where the displacement controlled loading method was selected for the ease of analysis of cyclic loadings. Because the stiffness of the reinforced concrete wall was seen to be sufficiently high until cracking had occurred.

4.2 Example

For an object of applying the present method of analysis, a reinforced concrete shear wall shown in Fig.4(a) was chosen. This part of the work was conducted in collaboration with Shioya and his associates of Shimizu Construction Co. The alternate compressive force is applied at both ends of the upper slab in a reversed cyclic mode. Special attention is paid to such defects as pull-out displacement of bars and shear slip of plane that take place at the junctions of columns and walls to the base slab, namely, those defects whose importance remained unrecognized in past studies by conducting precise measurements on them and by clarifying the effects they exert on the entire structural part.

The element discretization and boundary conditions employed in the present analysis are shown in Fig.4(b), where the presence, beside the reinforced concrete elements, of the joint elements that have been provided to account for deformations occurring in columns, slabs and walls, should be noted. In the joint element for the base slab to wall boundary, in particular, the effects of bond-slip of steel bars out from the base slab and those of local compression have been incorporated. Because the effects of apparent pulling out of bars out of walls have been dealt with in the calculation of mean strains of the reinforced concrete plate element, however, they are not included in the joint element. Since the model for bond-slip of steel bars (Shima et al, 1987) adopted in the present analysis is the one that was obtained under conditions of preventing the occurrence of vertical cracks and eliminating the effects of reduction in bond in the vicinity of loading ends, these effects need be incorporated when reduction in bond is expected to occur. In the present analysis, this was taken care of, for simplicity's sake, by doubling the value of slip given in the reference.

In the current form, the present example comprises 113 nodes, 20 plate elements, and 12 joint elements, and is loaded on a 16 bit personal computer. Since it takes about 3 to 8 minutes to solve one load step, tracing of a whole lifetime expended under monotonic loading that from generation of cracks to the ultimate load, can be performed in about 60 minutes of time.

Figure 4(c) presents the results, both analytical and experimental, in the form of the relation of the load to the horizontal displacement at the top point A. Agreement between the two is excellent both for the envelopes and for the internal history curves.

The ultimate load that the analysis gave was 202 ton against 201 ton of the experiment, while the ultimate displacement was 8.5 mm by the analysis versus 7.2 mm by the experiment.

The load to displacement relations at the leg (point B, 5 cm above the top surface of the base slab) are shown in Fig.4(d) for horizontal displacements and in Fig.4(e) for vertical displacements. The proportions of the slip at the junction of column and base slab in the overall horizontal displacement measured at point A were calculated to be 5% and 8% respectively for that at one half the ultimate load and that at full ultimate load. The proportion of the vertical displacement at the same place in the point A to the overall horizontal displacement were about 12% both at the one half the ultimate load and the full ultimate load. Since the proportion of the local discontinuous displacements at junctions of various components, such as the wall, columns and slabs are different, their effects on the apparent recovery or toughness of the wall as a whole should be included in the analysis.

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