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Seismic Performance of Large Underground Structures in Unsaturated and Liquefiable Soils

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ABSTRACT

Although seismic performance of large underground structures has been extensively studied recently, the focus of these researches is on the behavior of soil during earthquake motion, considering simple assumptions for the structure. The present study tries to have a more precise view towards the performance of large underground structures subjected to earthquake excitations. To achieve that aim, a typical subway tunnel is analyzed using dynamic Finite Element method coupled with soil and pore water. Both linear and nonlinear elements are used to model the underground structure. The computational results prove that nonlinearity of the structure should be considered to realize the accurate deformation and internal force response. Furthermore, the simulation is carried out in both drained and perfectly undrained conditions to clarify the effect of liquefaction on seismic response of the structures. It shows that ground liquefaction greatly deteriorates the soil skeleton stiffness and results in the reduced damage to RC structures even though large displacement of the ground is produced. It implies that the required ductility of RC ducts can be moderated provided that the liquefaction risk is high.

INTRODUCTION

Underground facilities as an integral part of the infrastructure of modern society are used nowadays for a wide range of lifeline applications, ranging from small pipelines such as those used in natural gas transmission to large underground structures including subway and highway tunnels usually made of reinforced concrete. The importance and cost of large-scale underground reinforced concrete infrastructures make it necessary to analyze the seismic behavior of underground structural systems including surrounding soil media accurately.

Although the seismic performance of large underground structures has been extensively studied (e.g. [Shawky and Maekawa, 1996](#), [Hashash *et al.* 2001](#) and [Huo *et al.*, 2005](#)), there have been limited researches regarding the liquefaction-related seismic performance of RC structures. [Liu and Song \(2005\)](#) investigated the dynamic behaviors of a subway station in liquefiable sand subjected to horizontal and vertical earthquake excitations. [Kimura *et al.* \(1995\)](#) conducted some centrifuge model tests to study the effect various countermeasures against liquefaction of sand deposits with

an underground structure. In most of these researches, however, the focus was addressed on the behavior of soil itself during seismic motions and the up-lift rigid body motion of berried structures. The present study tries to have deeper views towards the inelastic performance of underground RC structures subjected to earthquake excitations considering the nonlinearity of both structures and soil foundations.

In addition, Seismic design actions for underground ducts are generally characterized in terms of forced displacement and/or mean strains imposed on the structure. The rational and practical approach is to implicitly consider the interaction of underground RC with surrounding grounds. First, free-field ground deformations due to a seismic event are estimated and second, the underground RC is designed to accommodate these deformations through fictitious soil spring. This approach is satisfactory especially when lower levels of shaking are anticipated or the underground facility is in a stiff medium such as rocks ([Hashash *et al.* 2001](#)).

In this paper, a subway tunnel having interaction with surrounding soil is analyzed using coupled dynamic Finite Element method. In order to realize how nonlinearity of the structure influences the response, both linear and nonlinear elements are used to model the underground structure. In order to determine the effect of liquefaction on seismic response of structures, both drained and perfectly undrained states of pore water are discussed. Liquefaction may greatly increase in the deformation of soil around structures, but at the same time, the stiffness and internal stress of soil are dramatically reduced, too. A question is raised, what is the resultant of both kinematics in RC nonlinearity?

NONLINEAR CONSTITUTIVE MODELS

Constitutive Model for Reinforced Concrete. A reinforced concrete material model has been constructed by combining constitutive laws for cracked concrete and that for reinforcement. The fixed multi-directional smeared crack constitutive equations ([Maekawa *et al.*, 2003](#)) are used as the relations of spatially averaged stresses and strains. Crack spacing or the density and diameter of reinforcing bars are implicitly taken into account in smeared and joint interface elements no matter how large they are.

The constitutive equations satisfy uniqueness for compression, tension and shear of cracked concrete. The bond performance between concrete and reinforcing bars is taken into account in terms of tension stiffening and the space-averaged stress-strain relation of reinforcement is assumed to represent the localized plasticity of steel. The hysteresis rule of reinforcement is formulated based upon Kato's model for a bare bar under reversed cyclic loads. This RC in-plane constitutive modeling has been verified by member-based and structural-oriented experiments. Herein, the authors skip the details of the RC modeling and refer to [Maekawa *et al.*, 1997](#), [Maekawa and An, 2000](#) and [Kato, 1979](#).

Constitutive Model for Soil. A nonlinear path-dependent constitutive model of soil which can predict the nonlinear response of layered soil under earthquake excitation is essential to simulate the behavior of the entire RC–soil system properly. Here, the

multi-yield surface plasticity concept (Maekawa *et al.*, 2003) is applied to formulate the stress-strain relationship of the soil following Masing's rule for the shear hysteresis (Masing, 1926). The authors also use the framework of elasto-plastic and damaging concrete modeling to formulate the soil nonlinearity as follows.

The basic idea of this method is rather simple. First, the total stress applied on soil particle assembly, denoted by σ_{ij} , can be decomposed of deviatoric shear stresses (s_{ij}) and compressive mean pressure (p) as,

$$\sigma_{ij} = s_{ij} + p\delta_{ij}$$

where δ_{ij} is Kronecker's delta symbol.

Soil is idealized as an assembly of finite numbers of elasto-perfectly plastic components, which are conceptually connected in parallel. Each component is given different yield strength, so all components yield at different total shear strains, which results in a gradual internal yielding. Thus, the nonlinear behavior appears naturally as a combined response of all components. Hence, the authors propose the total shear stress carried by soil particles being expressed with regard to an integral of each component stress as,

$$s_{ij} = \sum_{m=1}^n s_{ij}^m(\varepsilon_{kl}, \varepsilon_{kl}^m, G^m, F^m)$$

$$ds_{ij}^m = 2G_0^m de_{ij}^m = 2G_0^m (de_{ij} - de_{p_{ij}}^m)$$

$$de_{p_{ij}}^m = \frac{s_{ij}^m}{2F^m} df$$

$$df = \frac{s_{kl}^m de_{kl}}{F^m} = \frac{s_{kl}^m d\varepsilon_{kl}}{F^m}$$

where G_0^m is the initial shear stiffness of the m -th component, and F^m is the yield strength of the m -th one. These component parameters can be uniquely decided from the shear stress-strain relation (Maki *et al.*, 2005).

In general, the volumetric components may fluctuate and affect the shear strength and stiffness of soil skeleton. In fact, the shear strength of soil may decay when increasing pore water pressure brings reduced confining stress to soil particle skeletons. The multi-yield surface plastic envelope may inflate or contract according to the confinement stress as shown in Figure 1. It can be formulated by summing up the linear relation of the shear strength and the confinement stress as,

$$F^m = \chi F_{ini}^m$$

$$\chi = \frac{(c - I_1' \tan \phi)}{S_u}$$

$$I_1' = \frac{(\sigma_1' + \sigma_2' + \sigma_3')}{3}$$

where, F_{ini}^m is the multi-surface plastic envelope, χ is the confinement index, (c, ϕ) are cohesive stress and frictional angel, and S_u is the maximum shear strength.

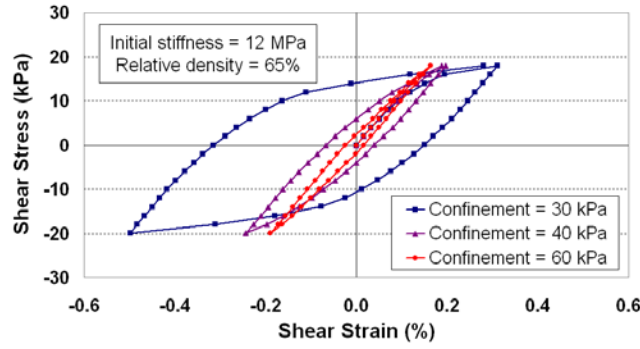


Figure 1. Confinement dependent soil model under drained cyclic shear loading

For simulation of the pore-water pressure and related softening of soil stiffness in shear, the volumetric nonlinearity of soil skeleton has to be taken into account. The authors simply divide the dilatancy into two components according to the microscopic events of soil particles. One is the consolidation or negative dilation as unrecoverable plasticity denoted by ε_{vc} . The other is the positive dilatancy associated with alternate shear stress due to the overriding of soil particles, which is denoted by ε_{vd} as,

$$p = 3K_0(\varepsilon_0 - \varepsilon_v), \quad \varepsilon_v = \varepsilon_{vc} + \varepsilon_{vd}$$

where K_0 is the initial volumetric bulk stiffness of soil particles assembly.

The volumetric irreversible contraction of particle will cause increasing pore pressure under hardly undrained states, which may lead to liquefaction. According to experiments of sandy soils, the following formulae are adopted as,

$$\varepsilon_{vc} = \varepsilon_{v,lim} \left\{ 1 - \exp(-2(J_{2p} + J_{2p,ini})) \right\} - \varepsilon_{vc,ini}$$

which is represented by the accumulated shear of soil skeleton denoted by J_{2p} (Maekawa *et al.*, 1997, Maki *et al.*, 2005) and $\varepsilon_{v,lim}$ is the intrinsic volumetric compacting strain corresponding to the minimum void ratio as,

$$\varepsilon_{v,lim} = 0.1(\log_{10} I_1^{0.6} + 1.0)$$

$$\varepsilon_{vc,ini} = \varepsilon_{v,lim} \left\{ 1 - \exp(-2J_{2p,ini}) \right\}$$

If the relative density of soil is assumed to be D_r , the following relation can be used to inversely decide $J_{2p,ini}$, which is a constant corresponding to the initial compactness of soil particles as,

$$D_r(\%) = \frac{\varepsilon_{vc,ini}}{\varepsilon_{v,lim}} = \left\{ 1 - \exp(-2J_{2p,ini}) \right\}$$

The shear provoked dilatancy which is path-independent and defined by the updated shear strain intensity denoted by J_{2s} as below,

$$\varepsilon_{vd} = \eta \frac{(aJ_{2s})^2}{1 + (aJ_{2s})^2}$$

$$J_{2s} = \sqrt{\frac{1}{2} e_{ij} e_{ij}}, \quad \eta = \frac{0.015(\varepsilon_{vc} + \varepsilon_{v,ini})}{\varepsilon_{v,lim}}, \quad a = 25.0$$

According to the elasto-plastic and continuum damaging model of concrete (Maekawa *et al.*, 2003), equivalent plasticity can be represented in general form with respect to the elastic scalar function as,

$$J_{2p} = \int dJ_{2p}^m, \quad dJ_{2p}^m \equiv \frac{1}{2} \bar{s}_{kl}^m d\varepsilon_{kl}$$

Then, the dilatancy factor can be defined in each component with different plastic range. Within this scheme, the liquefaction induced nonlinearity and cyclic dilatancy evolution can be consistently computed. Figure 2 shows the pure shear stress-strain relation and the corresponding pore pressure of undrained soil. Shear stiffness decay and cyclic mobility can be seen.

The overall experimental verification of the interaction analysis with soil and RC ducts was reported in (JSCE, 2002).

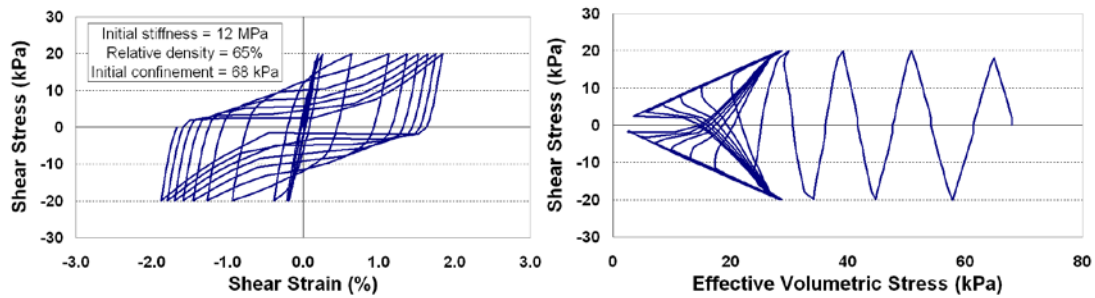


Figure 2. Confinement dependent soil model under undrained cyclic shear loading

Constitutive Model for Interface between RC and Soil. In this paper, the linear elastic model which assumes a bilinear relation for the opening/closure mode is employed to model the interfacial kinematics. The normal stress is zero in case of separation, which means no stress is transferred between the soil and the structure when the interface is open. On the other hand, the contact stiffness in closure mode is assigned a large value to ensure that no overlap is allowed, as shown in Figure 3(a). For shear sliding mode, shear stress–slip relation is assumed to be linear-plasticity as shown in Figure 3(b). The contact may slide if the applied shear stress exceeds the frictional shear strength, which is assumed to follow the Coulomb law. To apply this model, the initial condition of the soil–structure interface must be simulated to represent the actual static earth pressure. This is achieved by applying the natural gravity action of the soil mass alone before applying the dynamic action of the base rock (Maekawa *et al*, 2003, Maki *et al.*, 2005).

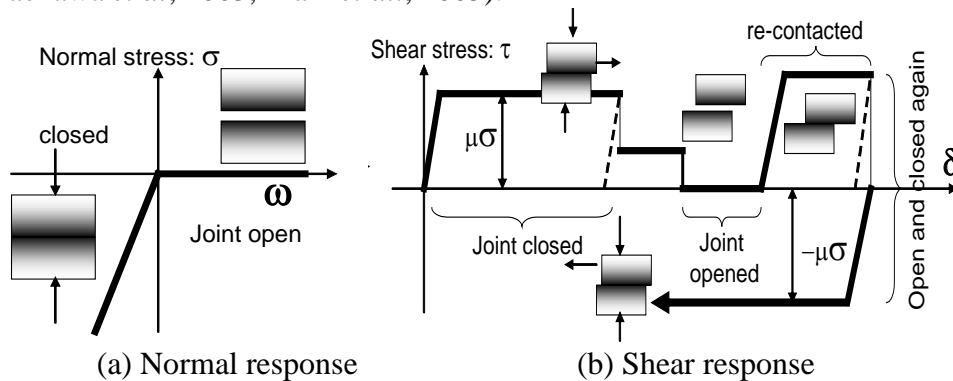


Figure 3. Normal and shear response of linear elastic interface model (Maekawa *et al*, 2008)

FINITE ELEMENT MODEL

Model Properties. To investigate the seismic behaviors of underground RC ducts, a typical subway tunnel section is modeled whose structural dimensions are shown in Figure 4. The center column to mainly support the dead weight of soil overlay has a rectangular cross section of $0.60 \times 0.80 \text{ m}$ and is idealized as firmly fixed to the slabs. The clear distance between two adjacent columns along the line is 3 m . The tunnel is stiffened with 45° haunches at the corners and has a longitudinal reinforcement ratio of 1.1% for side walls and slabs, 1.6% for the column, and web reinforcement ratio of 0.2% for all elements as shown in Figure 4.

The soil deposit is assumed to be loose sand with a friction angle of 30° and thickness of 15 m which is located on a 5-meter-thick layer of non-liquefiable soil which again lies on the bedrock as shown in Figure 4. The details of material property for reinforcing bars, concrete, interface joint and non-liquefiable soil layer are shown in Table 1.

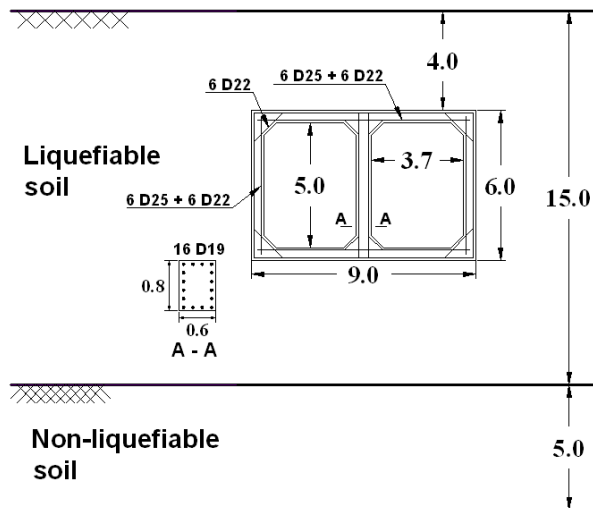


Figure 4. Soil-structure system geometry

Table 1. Material property

Non-liquefiable layer		Reinforced Concrete		Interface	
G_0	105 MPa	f'_c	24 MPa	Normal modulus	108 kPa
SPT N -value	15	Unit weight	24 kN/m ³	Shear modulus	103 kPa
Dry unit weight	16 kN/m ³	Poisson ratio	0.18	Friction angle	21°
Friction angle	40°	E_{steel}	2.0×10^5 MPa	Cohesion	0
Cohesion	100 kPa	f_y	240 MPa		
D_r	75 %				

By assuming the plane strain condition, the finite element mesh used in the analysis is composed of eight-node isoparametric two-dimensional elements for both RC and soil. The RC-soil interfacial elements are placed at an interface in between the soil and the RC elements. Since the angle of internal friction of the model sand is 30° , the friction angle of the interface is obtained using the relation $\delta = \tan^{-1}[(2/3) \tan \phi]$, which is about 21° . Totally, 7303 nodes and 2352 elements are arranged in the dynamic model. The north-south component of the rock base acceleration measured at 1995 Kobe earthquake, which is scale-adjusted to 0.3g based on the measurement at Kobe meteorological observatory, is used as the input bed rock motion in the seismic analysis. It shows a high horizontal ground acceleration with a short period as shown in Figure 5.

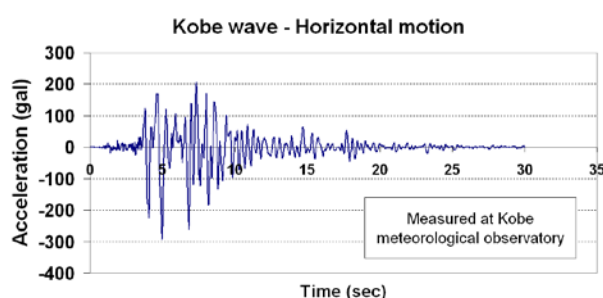


Figure 5. Input earthquake motion

Boundary Conditions. The boundary between the soil deposit and the bedrock is simply assumed to be fixed and would act as the bottom boundary of the analyzed domain at which the earthquake motion is imposed. The ground surface is assumed to be flat and free of loadings and the underground water level is assumed to locate up to the ground surface level when the soil is saturated.

The seismic behavior of the soil deposit in the far fields of the underground structure should assume the response of a free field. In the shaking table tests of soil-structures, a laminar shear box may be used to simulate the quasi-far-field boundary (Towhata, 2008). Here, quasi-far-field elements with a length of 10 m are placed at each extreme side of the analysis domain. The stiffness and unit weight of these elements are increased 100 times with respect to adjacent soil elements of the domain. As the far-field mode of seismic motion is simple shear, the length of the boundary condition (about the half of the domain height) is selected so that the bending deformation mode would not occur. In addition, confinement independent soil elements are used in the quasi-far-field zone in order to prevent the edge collapse in analysis. This boundary allows the harmonized horizontal and vertical displacements similar to the case of laminar shear box.

Several trial analyses were conducted to determine the size of the analyzed domain. Finally, a relative large analyzed domain (200 m) is used to make the reflected wave too weak to affect the calculated response in the focal part of interest to the authors.

Analytical Approach. The seismic analyses of the soil-structure system require that an initial stress field in equilibrium be obtained beforehand (Liu and Song, 2005,

Maekawa *et al.*, 2003). Therefore, an initial static drained analysis was firstly performed to determine the initial stress field and static earth pressure on the duct. This static stress field is then used as the initial condition for the subsequent dynamic run with the input excitation. The geological and construction history or path-dependence of the soil-structure system is not perfectly considered. But, the authors consider that these initial stress states may not be serious because of the high inelastic plasticity which is induced to soil under large ground motions.

In order that the modifying effect of the RC nonlinearity on the soil-structure response can be understood, the before mentioned tunnel is analyzed using linear elastic elements with a young modulus of 30000 MPa, Poisson's ratio of 0.2, and unit weight of 24 kN/m³, keeping the sandy soil property constant ($G_0 = 35$ MPa). The results are compared with those of nonlinear RC tunnel.

Then, in order to investigate the effect of soil liquefaction on the damage of underground RC ducts, several models with and without the duct are analyzed in both drained and undrained states of pore water with various initial stiffness which is increased from 12 MPa to 230 MPa as shown in Table 2. The structure is assumed to be located inside the soil at a depth of 4 m without any change in the mesh property of the remaining soil elements.

Table 2. Material property for first soil layer

SPT N -value	G_0 (MPa)	D_r (%)
1	12	25
3	29	28
5	44	32
10	76	40
15	105	48
20	132	56
25	158	65
30	182	75
40	230	80

It should be pointed out that a fully undrained condition is assumed for saturated soil elements during the seismic action which could be an extreme case but still a reasonably clear assumption, because the required time for drainage of a several-meter-thick sand layer is 10-30 minute which is much longer than the duration time of earthquake loading (Towhata, 2008).

NUMERICAL RESULTS

Excess pore pressure in the soil. The excess pore pressure responses of the saturated sandy layer with and without RC, at two different levels (above and below of the tunnel) considered at the centerline of the domain are shown in Figure 6. The excess pore pressures are expressed in terms of the ratio of excess pore pressure to the initial effective overburden pressure. It can be seen that the degree of liquefaction below the duct is lower with the presence of duct. In fact, the flotation of the

underground lightweight structure would cause larger shear deformation of soil which contributes to the lowering of excess pore pressure. This is reasonable for most medium loose and medium dense sand, since in larger shear deformation, the sand tends to dilate, which shall lead to the lowering of excess pore pressure if it is saturated (Liu and Song, 2005).

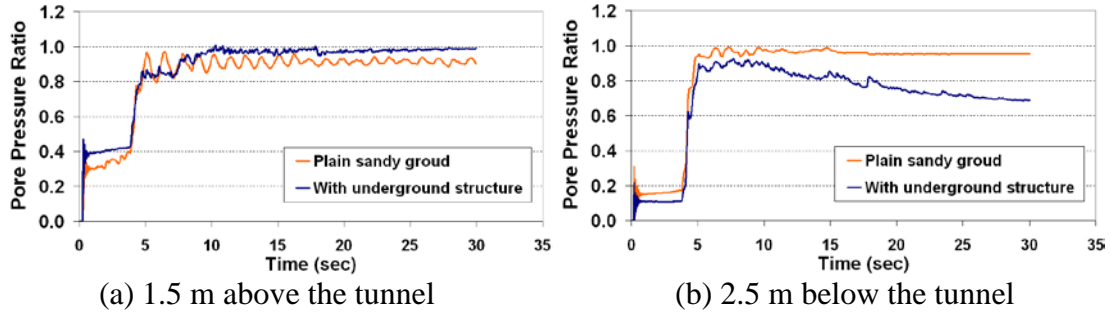


Figure 6. Excess pore pressure in soil

Effect of RC nonlinearity on underground structure response. In most of the researches regarding seismic response of underground structures, the researchers tend to model the structure with linear elastic elements. In order to understand the accuracy of this assumption, two analyses are carried out considering linear elastic elements for the tunnel in dry and saturated soil conditions. The vertical displacement response of the tunnel is shown in Figures 7. It can be observed that the duct would have some settlement during the ground motions in drained condition, while liquefied soil would push the underground duct upward significantly as it has been observed in the past earthquakes (Towhata, 2008). Therefore, some countermeasures should be considered to reduce the uplift of underground structure in liquefiable soils.

Furthermore, it can be said that the overall estimation of rigid body motion response of the structure in linear case agrees well with that in nonlinear case. The shear strain of the linear tunnel which deals with the deformation mode of the structure, however, is much smaller than that of nonlinear case as demonstrated in Figure 8. In fact, RC nonlinearities cause the duct to accommodate more shear deformation. Besides, since the linear tunnel remains stiff during the earthquake motion, the adjacent dry soil movement is restricted by the structure, the shear modulus degradation of the soil is limited which decreases the displacement demand on the structure (Huo *et al.*, 2005). In addition, the shear force response of the center column follows the shear strain response trend; i.e. the shear force of the linear column is smaller than that of nonlinear case as shown in Figure 9.

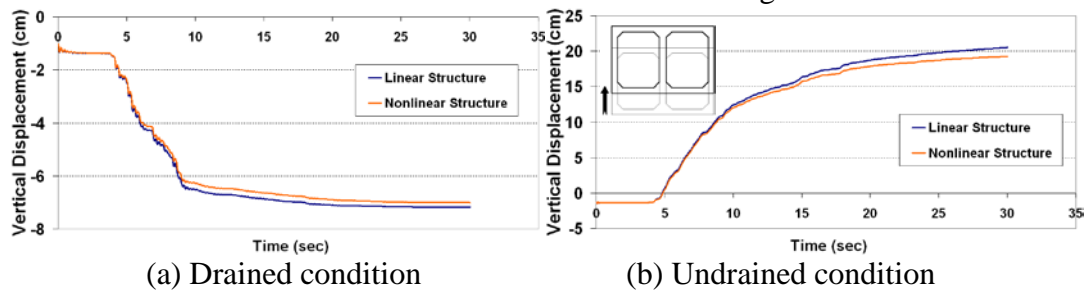
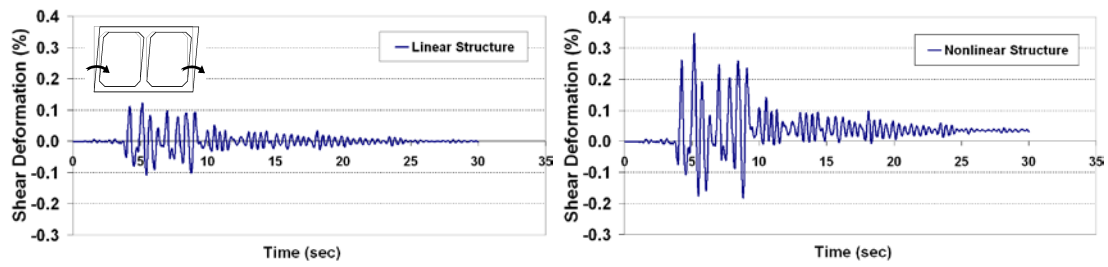
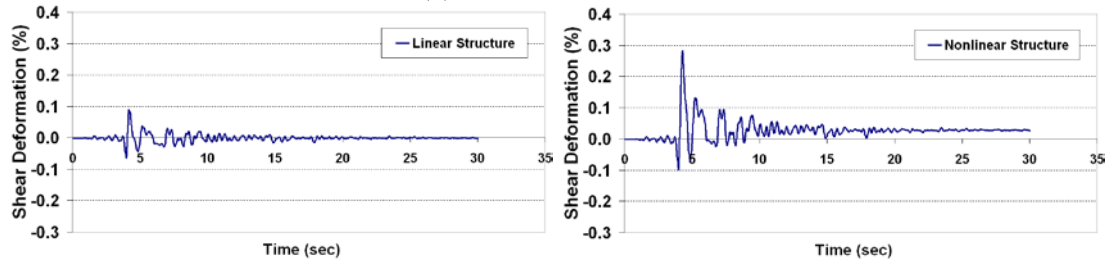


Figure 7. Vertical displacement of the tunnel



(a) Drained condition



(b) Undrained condition

Figure 8. Shear deformation response of the tunnel

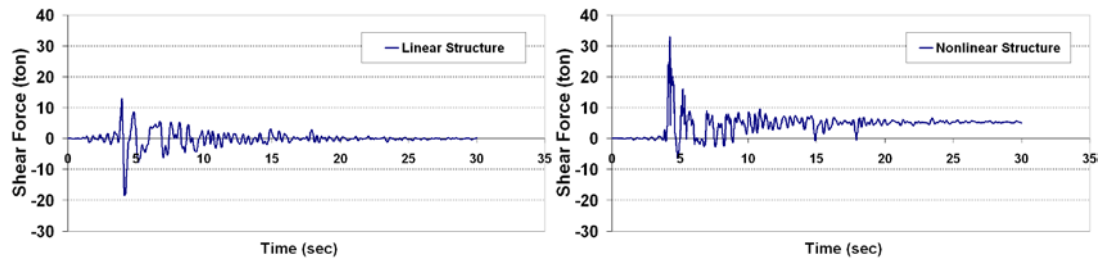


Figure 9. Shear force response of the center column in undrained condition

Effect of liquefaction. Figure 10 shows the maximum shear deformation response spectrum of the RC duct and the soil volume which replaces it in the plain sandy grounds. It can be observed that the on-going practical design approach based on the free-field ground deformations can well predict the deformational demand on the underground RC structures in somehow stiffer soil mediums. In soft layers of soil which consists of loosely deposited sand, however, the design based on the free-field ground deformations would result in an overestimated deformational demand on the structural members as shown in Figure 10(a).

Besides, liquefaction may significantly bring about increased soil deformation which indicates that large soil strains with associated large degradation of the shear stiffness have developed within the ground. Hence, the underground RC duct which is located in the regions where the underground water level is high and designed without considering the interaction with the surrounding ground results in large amount of web reinforcement or large dimensions of structural elements for ductility demand.

However, because of the deterioration of the surrounding soil stiffness which takes place in liquefied soil, the deformation demand on the RC duct would dramatically decrease resulting in less damage to the tunnel as shown in Figure 10(b).

Therefore, that large amount of reinforcement or thick structural elements is not necessary.

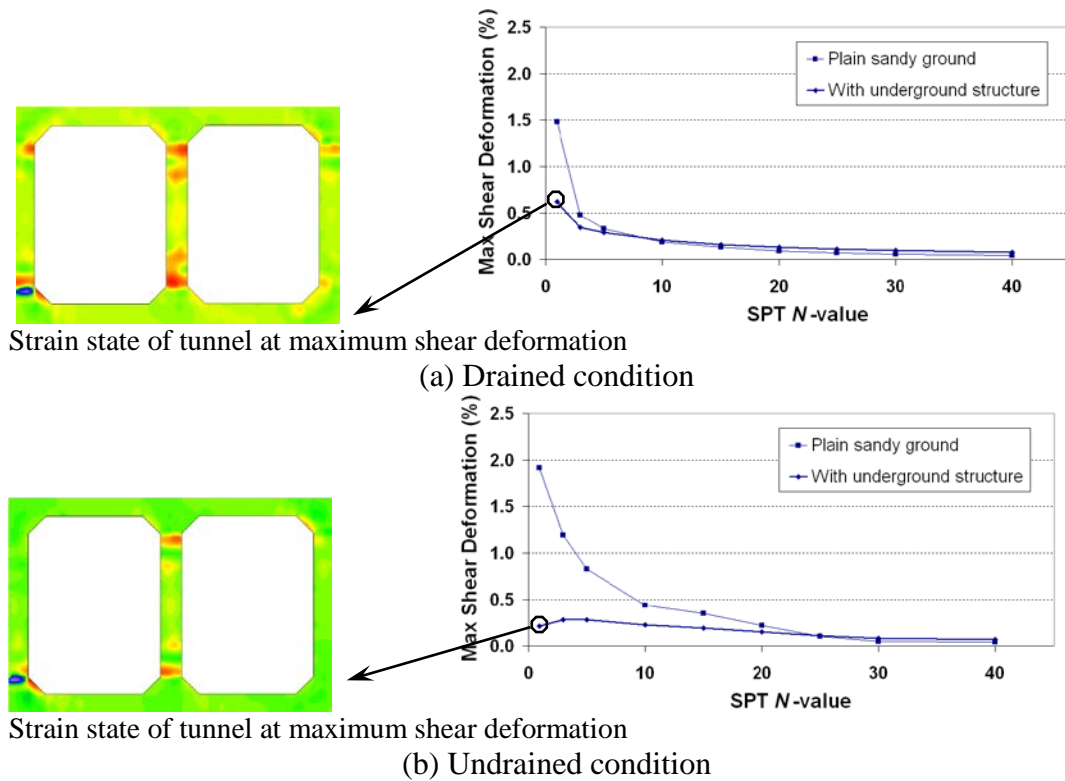


Figure 10. Effect of soil stiffness on shear deformation

CONCLUSION

In this paper, the nonlinear seismic response of an underground structure in dry and saturated liquefiable soils was investigated. From the numerical analyses, it can be concluded that assuming linear elements for underground structures in seismic analysis may well estimate the rigid body motion response of the structure; however, nonlinearity of the structure should be considered to realize the accurate deformation and internal force response.

Furthermore, in soft layers of soil which consists of loosely deposited sand, the design based on the large free-field ground deformations would result in high deformation demand on the structural elements which in turn requires large amount of reinforcement or large dimensions of structural elements. This situation would become even more severe in the regions where the underground water level is high because liquefaction may occur, which significantly increases the ground deformations.

However, liquefaction would deteriorate the surrounding soil stiffness and thus the deformation demand on the tunnel would consequently decrease. Considering this issue could lead to a more optimum, economical and rational design of the RC underground structures. Finally, it should be pointed out that other

countermeasures like sheet piling or increasing the weight of tunnels should be considered to reduce the uplift of underground structure in liquefiable soils.

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