

## 論文 Determination of Buckling Length of Reinforcing Bars Based on Stability Analysis

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**ABSTRACT:** Average compressive stress-strain relationship of reinforcement is sensitive to buckling length. The aim of this study is to propose an analytical method to predict the buckling length of longitudinal bars restrained by lateral ties inside RC structures. Stability analysis is conducted giving due consideration to both geometrical and mechanical properties of the longitudinal and lateral reinforcements. The required tie stiffness is derived from energy principle and compared with actual stiffness to determine the stable buckling mode.

**KEYWORDS:** energy principle, buckling length, stability analysis, tie spacing, tie stiffness

### 1. INTRODUCTION

Reinforcing bars, when subjected to axial compression exhibits large lateral deformation (hereafter referred to as buckling) especially after the absolute compressive strain becomes higher than the yielding strain. One of the most important parameters that govern the stress-strain relationship [1] of reinforcing bars in compression is buckling length to bar diameter ratio. In the compression tests of bare bar, the buckling length is equal to the supported length of the test piece. But for the reinforcing bars inside reinforced concrete members, this definition of buckling length does not apply. Hence, the determination of length to diameter ratio, in such cases, becomes difficult and requires proper consideration of interrelated mechanisms between main bar and lateral ties.

Previous researchers [2, 3, 4] have come up with different conclusions regarding the buckling length, varying from one to several times tie spacing. It is realized that the buckling length may extend to several times tie spacing depending on the arrangement and strength of lateral ties. However, if the size and spacing of the lateral ties are designed properly so that the stiffness of the stirrup is high enough to provide a rigid support to the longitudinal bar, it is ensured that the main reinforcement buckles between two adjacent stirrups. But, this is not always the case. Here, an analytical method to determine the buckling length is proposed.

### 2. ASSUMPTIONS AND GENERAL FLOW OF COMPUTATION

Longitudinal reinforcing bars are simulated as flexural members fixed to the lateral ties at two extreme ends of buckling length. Moment curvature ( $M-\phi$ ) relationship of elastic

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flexural member is given by  $M=EI\phi$ , where  $EI$  is the flexural rigidity. Because of its nonlinear nature, the flexural rigidity of reinforcing bar in post-yielding region is not unique and the exact value can be obtained through microanalysis. For simplicity, average flexural rigidity is assumed to be half of the elastic flexural rigidity for normal strength reinforcing bars. The average flexural rigidity of reinforcing bar is also influenced by its yield strength. For example, in case of high strength bars, the associated plasticity is comparatively smaller and the secant stiffness is also higher (figure 1). Consequently, the average flexural rigidity increases with increase in yield strength and vice versa. Finally, considering the influence of yield strength as well, the average flexural rigidity is assumed as  $EI = E_s I / 2\sqrt{f_y/400}$ , where  $E_s$ ,  $I$  and  $f_y$  (MPa) are Young modulus, moment of inertia and yield strength, respectively.

To define the deformational shape of the longitudinal reinforcement, boundary conditions ensuring zero lateral displacement and no slope at the end springs, are assumed. To fulfill these boundary conditions, a cosine curve, normally used for deformation of fixed end column, is used as shown in figure 1. The lateral ties are simulated by discrete elastic springs. In reality, the lateral ties show elasto-plastic behavior and after reaching the yield strain, the stiffness of the tie is reduced nearly to zero. The ties around the middle of the buckling length are prone to undergo high tensile strain due to large lateral deformation of the longitudinal bars. To cope with these facts, the springs within the central half of the buckling length are eliminated from the system, as shown in figure 1, for accurate prediction of required stiffness of other elastic springs.

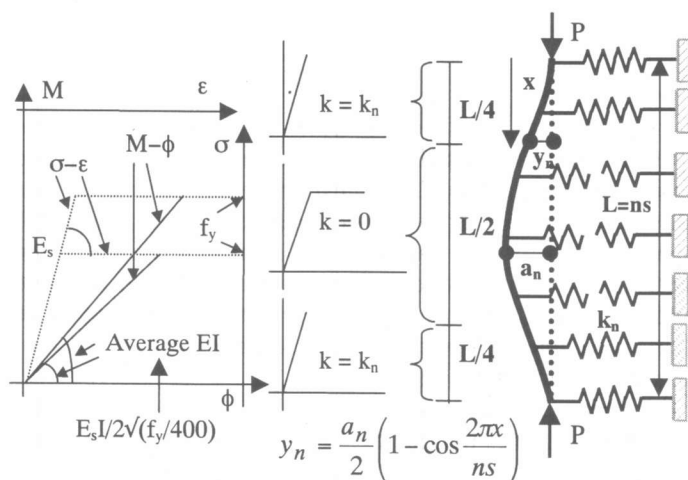


Figure 1. Simulation for stability analysis

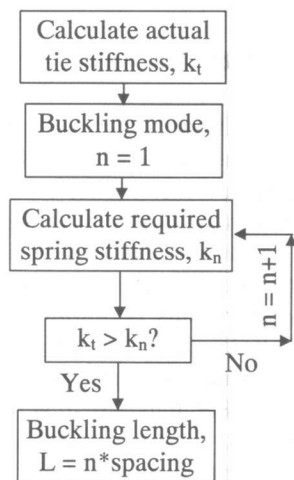


Figure 2. Flow-chart of entire process

The entire process of buckling length determination is illustrated with a flow-chart in figure 2. First, the actual tie stiffness effective to each longitudinal reinforcement is calculated. Next, the minimum spring stiffness required to hold the longitudinal reinforcing bars in different buckling modes is determined using energy principle, as stated later. For each buckling mode starting from 1, the required stiffness is compared with effective tie stiffness to check the stability of the reinforcements in corresponding buckling modes. The stable buckling mode is the smallest possible mode for which the required spring stiffness is less than the actual tie stiffness. The product of this stable buckling mode and the tie spacing gives the buckling length of the main reinforcement for the given arrangement of lateral ties.