

## 論文 Three Dimensional Modelling of Reinforced Concrete Members

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**ABSTRACT:** A three-dimensional (3D) constitutive model for nonlinear finite element (FE) analysis of reinforced concrete is presented. Concrete and reinforcement are combined to construct a composite finite brick element. The elasto-plastic and continuum fracture model is adopted for uncracked concrete. For post cracking formulation, fixed multi-directional, non-orthogonal smeared crack model is applied. A concept of 3D reinforced concrete (RC) zoning and anisotropy of strain softening/stiffening is discussed. Numerically obtained results are substantiated by experimental data.

**KEYWORDS:** 3D finite element analysis, anisotropic softening, reinforced concrete structures

## 1. INTRODUCTION

With continuing computer hardware development and with new findings in material research, the finite element method can be used for the simulation of experiments and the prediction of the structural performance of complex reinforced concrete. The world wide research effort of almost three decades led to the development of elaborate constitutive models of reinforced concrete for in-plane structures. At the same time only a few applications involving full 3D modelling of structural geometry, stress states and loading patterns have emerged in the literature.

In this paper, 3D constitutive models of reinforced concrete with special attention to post cracking formulations are introduced. While in 2D element-wise isotropic tension softening may be reasonable [1], in 3D anisotropy should be considered and is addressed here. For verification of the proposed three-dimensional constitutive relations short RC columns under multilateral loading in shear were selected. The load bearing mechanism becomes inherent three-dimensional and the problem can not be solved by 2D analysis. In fact, most reinforced concrete structures are subjected to complex load patterns in their live time. Short columns of elevated highways, for example, are exposed to axial load and uni-axial shear under serviceability conditions. However, in the case of earthquake or accidental impact, such columns have to sustain multi-axial shear and/or torsion. Then it is necessary to understand the complex behaviour of RC structures under multilateral loading pattern.

## 2. RC-BRICK ELEMENT

## 2.1 GENERAL CONCEPT

Reinforced concrete is idealised as a composite material consisting of concrete and reinforcement to be superimposed. By combining the constitutive laws of average stress and average strain for concrete and reinforcement [6], respectively the RC brick element has been constructed. In FE computation isoparametric brick elements with 20 nodes are used for reinforced concrete.

Concrete is, according to its state, treated in an un-cracked or a cracked concrete routine of the structural analysis frame. The generation of cracks is a branching point into nonlinearity and at the

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same time it marks the switching of concrete model of the proposed analysis concept. Crack initiation is decided by a simple Rankine criterion. Realising that the damage accumulation of concrete in compression and tension prior to cracking is basically of a non-local continuum nature, while post-cracking behaviour can be thought as a highly localised process, the separate treatment of un-cracked and cracked concrete seems to be reasonable. The phenomenon of post-peak compression softening remains un-addressed as outside of the present framework and a topic of further research.

## **2.2 ELASTO-PLASTIC AND FRACTURE MODEL OF UN-CRACKED CONCRETE**

In the pre-cracking range the triaxial elasto-plastic and fracture (EPF) model for concrete [4] is employed. Mechanical behaviours of concrete is idealised as combined plasticity and continuum fracture which identifies induced permanent deformations and the loss of elastic energy absorption capacity, respectively. The model, which originally was developed for full triaxial compression, had recently been extended to the whole 3D-space, covering tension domain of pre-cracked concrete [2].

## **2.3 3D SMEARED CRACK MODEL**

The introduction of cracks in concrete marks the transition from an idealised 3D continuum, treated by the EPF-model, toward a highly anisotropic medium. Owing to crack stress release, the 3D-confinement effect may be assumed to be much reduced for cracked concrete. Noting this inherent breakdown of 3D continuity, smeared crack modelling of three-dimension based on cracked concrete in-plane constitutive law had been proposed by Maekawa et al. [5]. The 3D strain field is decomposed into three sub-spaces of 2D so that the initially induced crack plane is parallel to one sub-space and the crack is contained in the other two 2D planes. The 2D reduced component stresses are computed based on mean strain projected on sub-spaces. Assuming the total load carrying mechanism as being composed of partial stresses on three sub-spaces the full 3D stress field of cracked concrete can be integrated. Since the two-way cracks are considered in each sub-space, triple cracks are fictitiously considered in the scheme of smeared crack idealisation. For computation of sub-space component stresses well established 2D constitutive models are available [6]. The crack analysis routine is composed of tension stiffening/softening model across cracks, compression softening model parallel to cracks and shear transfer model along cracks.

## **2.4 DISTRIBUTED REINFORCEMENT**

When we would use a discrete reinforcing bar model in concrete, much finer finite elements have to be allocated for numerically simulating localised yielding of reinforcement. But the smeared reinforcement model can take into account the localised plasticity by spatially average constitutive law of steel embedded in concrete with bond interaction. Then, for reinforcement orthogonal arranged in space distributed representation without dowel shear stiffness had been adopted. In the smeared approach localisation of initial steel plasticity close to cracks and bond slip effect are considered in computing the mean stress-strain relation based on the local bond-slip behaviour [6].

## **3. SOFTENING AND STIFFENING OF TENSIONED REINFORCED CONCRETE IN 3D**

### **3.1 RC-ZONING, SOFTEING AND STIFFENING MODELS**

To consider the different post-cracking behaviour of concrete close and far from reinforcing bars, RC-zoning is used. In the RC-zone, cracking is highly controlled by bond with reinforcement hence stiffening models are employed, while in the plain concrete zone sharp softening is assumed. The effective size of RC zone is influential on post cracking member stiffness and may even effect the failure mode. RC zone is determined by the condition that tension force carried by the RC zone just prior to cracking must be equal to the yield force which a reinforcing bar can support through bond