

# 論文 Tension Stiffness Modeling for Cracked Reinforced Concrete Derived from Micro-Bond Characteristics

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**ABSTRACT:** The aim of this study is to get the spatial average stress- average strain relationships of both reinforcing bars and cracked concrete in RC members based on the local bond characteristics between concrete and re-bars. The computational basis is the local bond-slip-strain model [3]. In the computation the local stress and strain profiles of both re-bars and concrete between two adjacent cracks are computed. Using these profiles, the spatial average stress and average strains can be computed. The computation is also capable of predicting the ultimate average strain of re-bars. The comparison with the experiments shows good agreement.

**KEY WORDS:** Bond-slip-strain, tension stiffening, crack spacing.

## 1. INTRODUCTION

The tension stiffening effect represents the capacity of the concrete to carry the internal tensile force developing between adjacent cracks. The tensile force is solely carried by the steel reinforcing bars at the cracked section. This force is transferred to the concrete between adjacent cracks through the bond stresses between reinforcing bars and concrete. The tension stiffening effect is usually treated by assuming a relationship between the average concrete tensile stress and the average concrete tensile strain over a long-gauge length in the direction normal to cracks. At the same time, the stress-strain relationship of reinforcement has to be on average basis. As the stress in reinforcement embedded in concrete vary along re-bars, the average stress-average strain relationship of reinforcement is significantly different from pointwise one of bare bar. The rebar begins to yield at the position of concrete cracks prior to the remaining parts of re-bars. Therefore, the average yield stress will be lower than the yield stress of the bare bar [4]. After yielding some parts of reinforcement close to crack will be in the strain hardening zone, whereas the remaining parts which are far from the crack will still in the elastic zone. Therefore, the average response has a stiffness mixed from the elastic and the hardening one. Usually, a bilinear model is assumed for the average response of steel bars. The aim of this study is to get the average stress-average strain relationship of reinforcing bars as well as for concrete using a reliable physical bond model.

## 2. SPATIAL AVERAGED CONSTITUTIVE LAWS IN TENSION

### 2.1. BOND-SLIP-STRAIN MODEL:

Shima et al. [3] proposed a universal bond stress-axial slip-steel strain model for RC . The model

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offers unique relationship which expresses the bond characteristics derived from both pull out and axial tension tests. The constitutive law of bond is given by,

$$\tau(\epsilon, s) = \tau_0(s) g(\epsilon) \quad (1)$$

$\tau(\epsilon, s)$ : Bond stress,  $\tau_0(s)$ : Bond stress when strain is zero

$$\tau_0(s) = f'_c k [\ln(1 + 5s)]^c \quad (2)$$

$$g(\epsilon) = \frac{1}{1 + 10^5 \epsilon} \quad (3)$$

$f'_c$ : Compressive strength of concrete,  $k$ : Constant=0.73,  $c$ : Constant=3

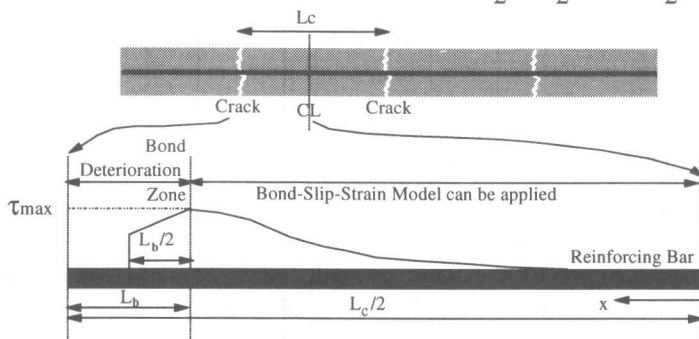
$s$ : Non dimensional slip =  $1000S/d$ ,  $S$ : Slip,  $d$ : Diameter of bar,  $\epsilon$ : Strain of bar

## 2.2. BOND DETERIORATION MODEL:

Shima's model can not be applied to the bond deterioration zone where the "near crack surface effect" is predominant. In fact, the localization of plastic yielding is initiated from the bond deterioration zone. Thus, the modeling of bond close to cracks plays an important role for post-yield behaviour of RC in tension. Qureshi et al.[5] assumed in the RC joint model that the bond stress is linearly decreasing to zero at a distance  $5d$  from the crack surface, and that the bond stresses drops suddenly to zero at a distance  $2.5d$  from the crack surface due to splitting and crushing of concrete around the bar beside the crack surface. **Fig.1** shows a schematic drawing of bond deterioration model. The model is,

$$\tau(x) = \tau_{\max} - \frac{\tau_{\max}}{L_b} \left\{ x - \left( \frac{L_c}{2} - L_b \right) \right\} \quad \left( \frac{L_c}{2} - L_b \leq x \leq \frac{L_c}{2} - \frac{L_b}{2} \right) \quad (4)$$

$$\tau(x) = 0 \quad \left( \frac{L_c}{2} - \frac{L_b}{2} \leq x \leq \frac{L_c}{2} \right) \quad (5)$$



**Fig.1** Bond Deterioration Model [5]

## 3. ANALYSIS

In order to get the steel stress profile, four equations should be solved simultaneously. By dividing the reinforcing bar between two adjacent cracks into small divisions or elements and studying the free body equilibrium of such elements, we get the following equilibrium equation,

$$\frac{d\sigma}{dx} = \frac{\pi d}{A_s} \bar{\tau} \quad (6)$$

where,

$\frac{d\sigma}{dx}$ : Re-bar stress gradient along an axis,  $A_s$ : Re-bar cross sectional area,  $d$ : Diameter of reinforcing bar,  $\bar{\tau}$ : Average bond stress

The second equation is the bond-slip-strain model, together with the bond model in the bond-deterioration zone. The third equation is the slip compatibility equation. The slip is computed by integrating the strain over the length of the rebar starting from the midway between adjacent cracks, i.e. the slip at the midway between cracks is zero. Thus, we have,

$$S = \int \epsilon dx \quad (7)$$

The fourth equation is the constitutive equation for the bare bar which represents the pointwise relationship between the re-bar stress and strain at each bar section,

$$\sigma = \sigma(\epsilon) \quad (8)$$

Firstly, the crack spacing is equal to the total length of the specimen, and during analysis the local concrete tensile stresses are checked and a new crack is introduced whenever the stress reaches the cracking stress of concrete and a new average crack spacing is computed. Starting from the midway between two adjacent cracks, a finite segment with length  $\Delta x$  is studied. The boundary conditions are assumed by equating both the slip and the bond stress at the middle section to zero, and assuming a value to the strain at the middle. The four equations are simultaneously solved using an iterative procedure. Firstly, strain at the outer end of  $\Delta x$  is assumed. Then, the problem is to compute  $\Delta x$  satisfying the equations. By assuming  $\Delta x$ , the slip at the outer end is computed using equation (7), then the bond stress is computed using equation(1). The average bond stress is then computed, and  $\Delta x$  is computed using equation (6). If the difference between the assumed  $\Delta x$  and the computed one is within the proposed accuracy, the assumed value is accepted. If it is far from the proposed accuracy,  $\Delta x$  is reassumed again, till getting the correct value. Thus, finishing the computation of this division, the boundary conditions of the next division are defined and a similar computation procedure is followed. Hence, the strain and stress profiles of the steel reinforcement can be drawn. It results in the average stress and average strain as,

$$\bar{\sigma} = \frac{2}{L_c} \int_0^{\frac{L_c}{2}} \sigma(x) dx \cong \frac{2}{L_c} \sum_0^{\frac{L_c}{2}} \sigma(x) \Delta x \quad (9)$$

$$\bar{\epsilon} = \frac{2}{L_c} \int_0^{\frac{L_c}{2}} \epsilon(x) dx \cong \frac{2}{L_c} \sum_0^{\frac{L_c}{2}} \epsilon(x) \Delta x \quad (10)$$

By computing the stress profile of reinforcement, the stress profile of concrete is obtained by subtracting the reinforcement force profile from the total force which equal to the re-bar force at the cracked section. Then, the average stress of concrete is also mathematically defined. A summary of the computation procedure is shown in **Fig.2**. A comparison with the experiments by Shima [3] is shown in **Fig.3** and **Fig.4**. The analysis agreed well with the reality .

#### 4. MEMBER ANALYSIS

As an application to the tension stiffening analysis, a FEM analysis was carried out to two cantilever columns tested under cyclic loading [7]. First, the average stress -average strain relationship of re-bars was computed. Then it was approximated to a bi-linear model composed of two lines, one is the elastic line up to the average yield point and the other is the strain