

# 論文 Path-Dependent Nonlinear Model of Reinforced Concrete Shells

Paulus IRAWAN\*<sup>1</sup> and Koichi MAEKAWA\*<sup>2</sup>

**ABSTRACT:** Geometrical nonlinear and material path-dependent constitutive models are presented for the analysis of reinforced concrete shells through finite element method. These models cover loading, unloading and reloading paths. Cracked reinforced concrete is modeled as an orthotropic material using smeared crack approach. Compression softening and tension stiffening effects are included in the derivation of constitutive equations. Layered formulation is used to discretize reinforced concrete shells in thickness. Analytical results are verified using data from experimental works under various loading conditions.

**KEYWORDS:** reinforced concrete shells, constitutive equations, finite element analysis

## 1. INTRODUCTION

To correctly understand the behavior of reinforced concrete shells and to predict its response under external loads, realistic constitutive laws for reinforced concrete are crucial. In the past various constitutive models have been proposed [1,2]. However, most of these constitutive models only deal with monotonic loading case where path-dependency is less important than the case of cyclic loading.

At present, path-dependent constitutive models for cracked concrete in compression, in tension, in shear, and a path-dependent model of reinforcement in reinforced concrete are available [3]. These constitutive models cover loading, unloading and reloading conditions and have been successfully used to predict the response of concrete panel under in-plane loads [3].

In this study, these models are incorporated through layered formulation to predict the behavior of reinforced concrete shells with combined in-plane and out-of-plane loads and also under cyclic loads through an efficient nonlinear finite element algorithm. Formulation of element and verification with test data are presented in this paper.

## 2. ELEMENT FORMULATION

Eight-node Serendipity isoparametric element with six degree-of-freedom in each node, three translations and three rotations, was used in the analysis. Reissner-Mindlin formulation was adopted to take into account shear deformation of concrete shells. The formulation is made based on the following assumptions:

1. Normals to the mid-surface remain straight but not necessarily normal to the mid-surface after deformation.
2. Stresses normal to the mid-surface are negligible.

---

\*<sup>1</sup> Department of Civil Engineering, The University of Tokyo, ME., Member of JCI

\*<sup>2</sup> Department of Civil Engineering, The University of Tokyo, DR., Member of JCI

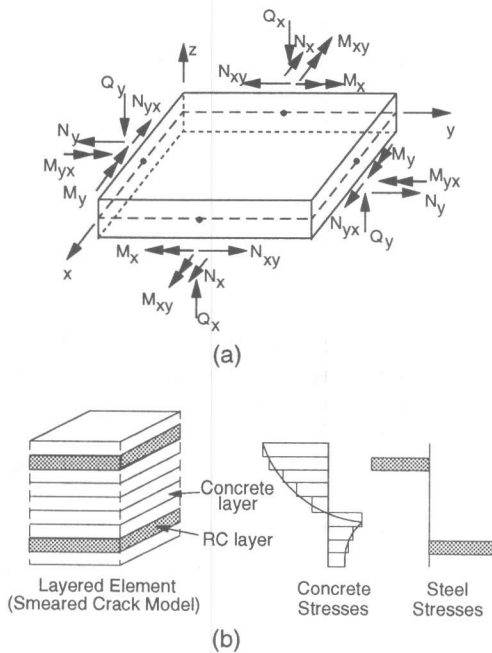


Fig. 1 Layered Element: (a) Forces Acting on RC Shells; (b) Plain Concrete and RC Layers

stiffening in this analysis and does not depend on the location from reinforcement. In other words, in this analysis tension stiffening used in every layer – with and without reinforcement – is similar and based on tension stiffening with reinforcement smeared in the whole concrete. Average stress-average strain relationship of steel reinforcement in concrete is also derived based on this assumption. Internal forces are calculated by integrating the corresponding stresses from each layer over the thickness of the element. Figure 1 gives the illustration of layered element and forces acting on the shells.

### 3. MATERIAL MODELING

A smeared fixed crack approach of multi-direction was incorporated in the material modeling of cracked concrete. It consists of tension stiffening model, compression model and shear transfer model. A path-dependent nonlinear model of reinforced concrete has been constructed by combining those models with a model of reinforcing bars in concrete. Prior to cracking, concrete is modeled as an elasto-plastic and fracture material with the introduction of fracture parameter as an indicator of the reduction of elastic modulus in the unloading process [3].

Tension model is independent of spacing of cracks, direction of reinforcing bars and reinforcement ratio. It is modeled in the form of average stress versus average strain of concrete. After average strain reaches cracking strain, concrete stress decreases gradually to take into account tension stiffening effect. In the reversed cyclic loadings, concrete stress is the sum of stress transmitted from the reinforcing bars and that transmitted from the closing of the cracks [3].

Compression model is based on elasto-plastic fracture model similar to the pre-cracking model. The effect of compression softening due to the present of transverse cracks, which causes the reduction

The use of exact numerical integration to obtain stiffness matrix of element tends to cause shear strain to impose constraints  $\gamma_{xz}=\gamma_{yz}=0$  in the total potential energy when the limiting thin shells are approached. This constraint is widely known as shear locking [4]. To avoid the locking in this analysis, numerical integration with the reduced second order of Gaussian quadrature was performed. The element has been tested to be free of shear-locking for the thickness of the specimens used in the verification. Geometrical nonlinearity was accounted by total Lagrangian formulation.

Shell element is divided into several layers of panel where in-plane constitutive models [3] were applied to take into account material nonlinearities as discussed in chapter 3. In the depth of its mid-surface, one integration point was used for each layer of panel. Each layer is classified as plain concrete or reinforced concrete layer with reinforcing bars being smeared in the layer. This classification is important to define the location of reinforcement in the calculation of internal bending moment of the shells. Since the area of concrete which actively contributes to tension stiffening is not known exactly, it is assumed that whole concrete contributes uniformly to the tension