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Development of Analytical Models for Reinforced Concrete

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Abstract: A computer program called WCOMR was developed to analyze the characteristics of RC walls subjected to reversed cyclic loading. The WCOMR comprises RC element idealization described by smeared crack model and joint element by discrete crack model. The analytical treatment of bi-directional crack, including the crack opening and closure, was explained based on the coordinate transformation. The nonlinear solution to satisfy the governing equations of equilibrium was presented. The applicability of the WCOMR has been experimentally verified by the element and structural levels.

Introduction

In making analytical prediction for inelastic behaviors of reinforced concrete subjected to reversed cyclic loading, modeling of reinforced concrete elements including the cracking habit, from yielding of the reinforcing bars and on through subsequent repetition of loading, is indispensable. As an RC wall, behaviors of cracks after they have been generated and attained a stable state are rather more important than the development of individual crack. For this purpose, the smeared crack model, in which a finite region containing several cracks and reinforcing bars are considered to be a continuum, is quite adequate to describe a reinforced concrete element.

On the other hand, reality is that local discontinuities, like pull-out of bars and intrusion of junction planes, can take place due to abrupt changes of rigidity induced at the joints of different structural elements of which the reinforced concrete wall is one as shown in Fig.1. To take these effects into account, introduction of the discrete crack model becomes necessary. By combining these two models, the authors have developed a computer program called WCOMR developed for FEM analysis of reversed cyclic response of reinforced concrete walls.

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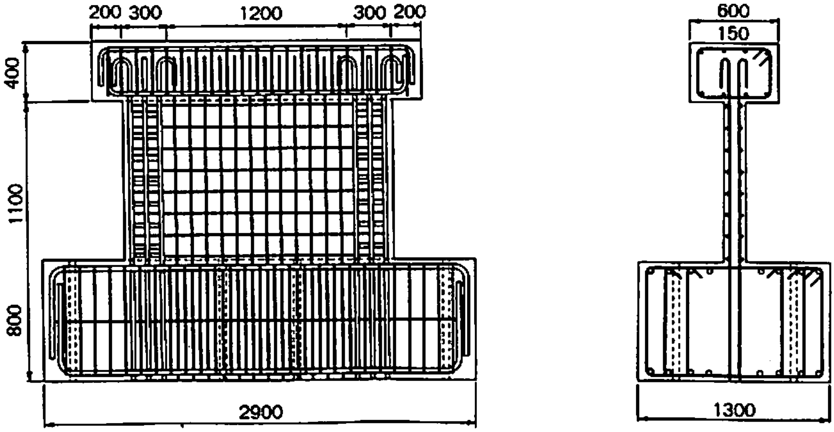

Fig.1 Reinforced concrete wall specimen

Development of Analytical Model

Although number of models for FEM analysis of reinforced concrete exist and their method of approach may differ, any model has a set of empirical formulae, the only difference being where the modeling for analysis has begun(Okamura, 1986). It is quite natural that the closer the analytical model has its origin to the problems, the greater is its accuracy. But, It is important to note that the closer to the level 1 in Fig.2 the model has been elected to originate, the wider the application it becomes able to claim. The authors have been developing a comprehensive analytical model starting off at the level 1 of Fig.2 to describe accurately the structural behaviors, including those under reversed cyclic loading.

Smeared Crack Model for Reinforced Concrete Elements

The smeared crack model for reinforced concrete elements falls on the level 2 in Fig.2. It may be constructed by combining micro-models in level 1. Care should be taken for the fact that judgment of its fitness must be done by comparison with experiments conducted on the reinforced concrete element level(Okamura. 1986) since experimental conditions for micro-models which we constructed are often limited and idealized. The smeared crack model was verified as capable of simulating both the empirically determined envelopes and internal history curves quite well as discussed in the companion paper (Izumo, 1989).

However, one deficiency with this model is its complexity since its micro-models are empirical formulae,

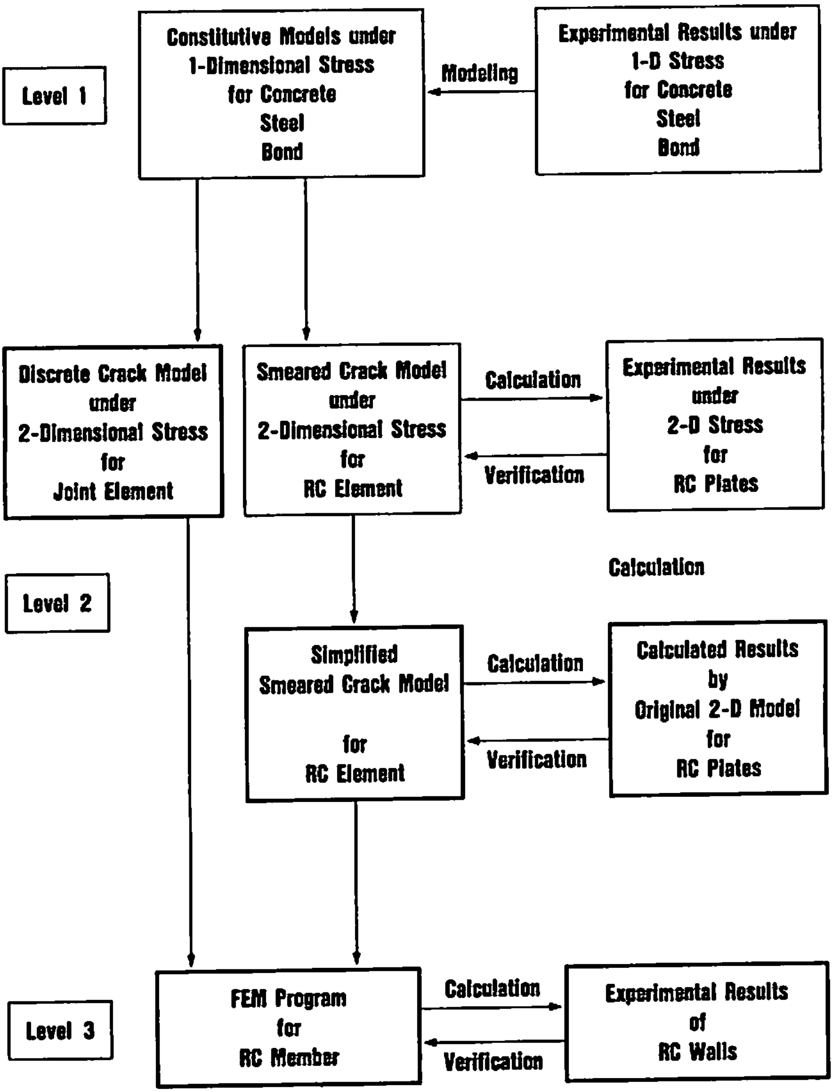

Fig.2 Development of analytical models and verification

each describing faithfully an individual behavior (Maekawa and Li, 1988 and Shima, 1987). For simplification to make FEM analysis possible, a series of level 2 analyses was conducted to develop a simplified smeared crack model applicable to level 3. The analytical results conducted for the walls are based on this simplified model.

Discrete Crack Model for Joint Planes

It is to be remembered here that local discontinuities, such as pull-out of steel bars and slipping or intrusion of junction planes, can and do often take place as a result of abrupt changes in the section stiffness occurring at the joint planes connecting two components of different thicknesses, and that many FEM analyses have given load-displacement relations that run higher to the observed ones because their influences are overlooked. Based on the discrete crack model in level 2, we have provided an indispensable joint element that describes the stress versus localized deformation relationship of the junction plane between two reinforced concrete elements of different sections.

Modeling for opening.- As the opening of discrete crack element represents the pull-out of reinforcing bars from the adjacent members with different thickness, the element behavior in opening mode can be described by the slip versus steel strain relation, which is applicable to both elastic and plastic states (Shima, 1987). This model is classified into level 1 in Fig.2. In simplifying the original model in the reloading path, we adopted second order polinominals for reversed cyclic model as shown in Fig.3. By combining constitutive model for steel bars under reversed cyclic load, stress versus opening relation for joint element was obtained.

Modeling for closure.- Based on two dimensional finite elements, it is impossible to take directly into account the localized strain distribution over the members' thickness as shown in Fig.4. Since ignorance of this three dimensional effect means the underestimated closure between members with different thickness, we took into account this additional displacement in the joint element by introducing an virtual volume equivalent to this three dimensional strain localization. The equivalent height of the virtual volume, which has the less thickness of connecting elements was determined by comparing its effect in two dimensional analysis with three dimensional one. In fact, the analytical contribution of this effect to the displacements of the wall shown in Fig.1 were at most 3%.

Modeling for shear slip.- The slip along a joint element must be the interface shear displacement plus the shear deformation of the virtual volume mentioned above. Micro-models for shear and normal compression transfer along a crack (Li, 1988) and concrete under biaxial stress states (Maekawa, 1983) in level 1 were incorporated. The slip cyclic response of the joint element under a fixed crack opening is shown in Fig.5.

As will be discussed later, one significance of clarifying the influences of local displacements at the junction planes in reversed cyclic FEM Analysis of reinforced concrete walls lies in the fact that analytical predictions become possible for real size structures, whose properties are difficult to be examined with reduced size test specimens because of the size effect.

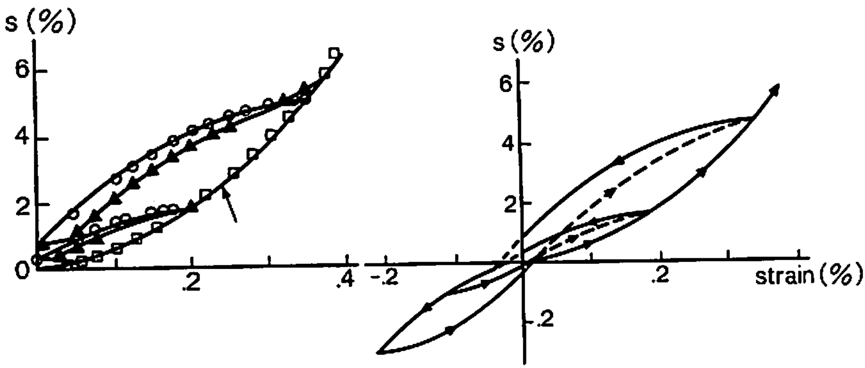

Fig.3 Modeling for opening of the joint element
(a) unloading and reloading (b) reversed loading

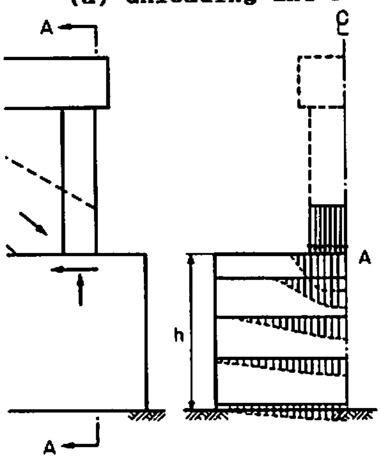

Fig.4 Stress distribution over the thickness

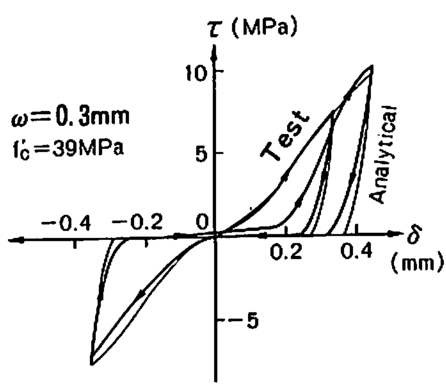

Fig.5 Modeling for shear slip of the joint element

Development of the Computer Program

The new finite element program was developed by incorporating the reinforced concrete element model and the joint element model described above into the program COMM2 (Maekawa, 1983) that we have had formulated for concrete member under monotonic loading and implemented previously.

Inelastic Solutions.- The numerical analysis is more complex compared with the case of monotonic loading. In the first place, the stresses on any element are obtained, not by the conventional step-by-step integration of the tangential stiffness, but by computing stresses directly from the total strain given by the constitutive relations. Therefore, the stiffness matrix used in the iterative calculation is for a mere sake of determining how to correct the assumed strains for the unbalanced forces present. In other words, the matrix is determined so as to make the calculation convergence most effectively as follows.

(1) No negative stiffness for iteration scheme shall be used even when the material tangential stiffness of element as a continuum has become negative. This is to prevent the iterative solution from diverging as the stiffness matrix becomes singular.

(2) The larger of the two stiffnesses shall be taken in the convergence calculation whenever the stress-strain relation changes. This is to suppress oscillation and divergence of solution in the iterative calculation.

(3) Judgment whether an finite element is under loading or reloading shall be done with reference to the structural loading condition. This insures taking of the correct one among the two or more stress solutions for a given strain, a state of affairs that can arise in any element model for cyclic loading.

(4) The constitutive relation of the element model shall conserve its continuity under any loading histories. That the continuity be conserved even at the inflection points in the loading history is one of the most fundamental and the most important among computational conditions for any element model to satisfy.

(5) In calculating stresses from strains, no implicit models that necessarily call for iterative operation to obtain stresses shall be used. This is to prevent the computation time from prolonging unduly because of the convergence calculations that are conducted for each of the elements, and because of the difficulty of efficiently obtaining the stiffness that is inherent in such methods.

(6) Even though the stiffness matrix is revised for each round of the iteration routine up to the fifth by which time such large changes in the stiffness as occurrence of cracks or of yielding of the steel bars should have occurred, no revision is to be made for further rounds like in the modified Newton-Raphson method.

Method of Dealing with Bi-Directional Cracking.- Stresses working in a reinforced concrete element can be calculated from the incidental strains in the model described above by performing coordinate transformation with reference to the crack axis on which the crack open and closure are dealt with (Maekawa, 1983). When a second crack has been generated, however, the reference axis is transferred from the first crack to the second. In this case, deformation at the closed crack is ignored. The stiffness of a reinforced concrete element is determined by the deformation occurring at the surfaces of the reference crack whose cracking width is larger, provided that the angle the two cracks make is not smaller than 15 deg. Subsequent change of the reference crack in any loading cycle is conducted when, and only when, the stress working normal to that crack concerned has become smaller than that for the other crack.

FEM Analysis of Reinforced Concrete Walls

For an object of applying the present method of analysis, a reinforced concrete shear wall shown in Fig.1 was chosen. This part of the work was conducted in collaboration with Dr. T. Shioya and his associates of Shimizu Construction Co. The alternate horizontal force is applied at both ends of the upper slab. Special attention is paid to such defects as pull-out of bars and shear slip of plane occurring at the junctions of columns and walls to the base slab, namely, those defects whose importance remained unrecognized in past studies by conducting precise measurements on them and by clarifying the effects they exert on the entire structural part.

The smeared and joint elements were arranged as shown in Fig.6(a). Because the effects of apparent pulling out of bars out of walls have been dealt with in the mean strains of the reinforced concrete plate element, joint elements only represent the pulling out of bars from the base slab. Since the bond model of steel(Shima,1987) adopted in the discrete crack model derived from conditions of preventing the splitting cracks and eliminating the bond deterioration near the loading ends. These effects need be incorporated when if reduction in bond is expected to occur. In the present analysis, this was taken care of, for simplicity, by doubling the slip value given in the reference.

Since it takes about 3 to 8 minutes to solve one load step with 16 bit personal computer, tracing of a whole lifetime expended under monotonic loading up to the ultimate load can be performed in about 60 minutes of time.

The multiplied deformation mode is shown in Fig.6(b), and analytical and experimental relations of the load versus the horizontal displacement at point A in Fig.6(a).

Agreement between the two is excellent both for the envelopes and for the internal history curves. The load to displacement relations at the leg are shown in Fig.6(d),(e) for horizontal and vertical displacements.

The proportions of the slip at the junction of column and base slab in the overall horizontal displacement measured at point A were calculated to be 5% and 8% respectively for that at one half the ultimate load and that at full ultimate load. The proportion of the vertical displacement at the same place in the point A to the overall horizontal displacement were about 12% both at the one half the ultimate load and the full ultimate load. Since the proportion of the local discontinuous displacements at junctions of various components, such as the wall, columns and slabs are different, their effects on the apparent recovery or toughness of the wall as a whole should be included in the analysis.

As an extreme case of experimental verification, another shear wall, which has the same dimension as the steel reinforced concrete wall as shown in Fig.1 but reinforced only by the glass-fiber mesh, was tested. This new fiber mesh has as low elasticity as concrete, no plasticity but approximately two times higher tension-stiffening than deformed steel bars. The effect of these material properties in level 1 to the cyclic structural response was successfully predicted by the WCOMR.

Furthermore, the authors examined the applicability of the program WCOMR with reference to the beam and column cyclic behaviors, which were collected by JCI as its recommendation (JCI Committee, 1983) for the check of analytical models. The details of these examination in level 3 will be reported in the near future.

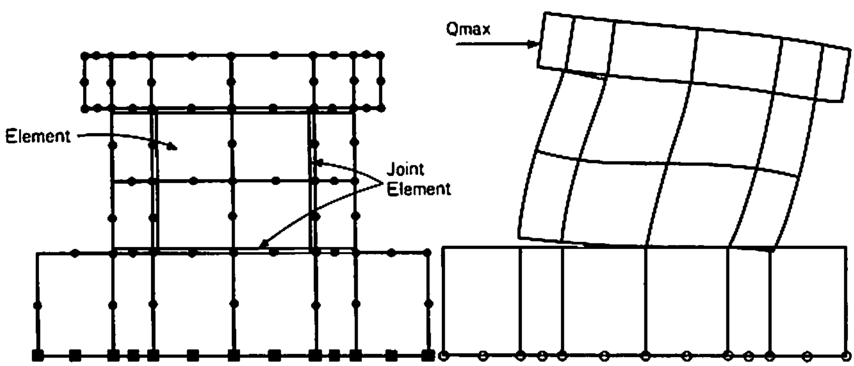

Fig.6(a) Finite element mesh (b) multiplied deformation
113 nodes, 32 elements mode of RC wall

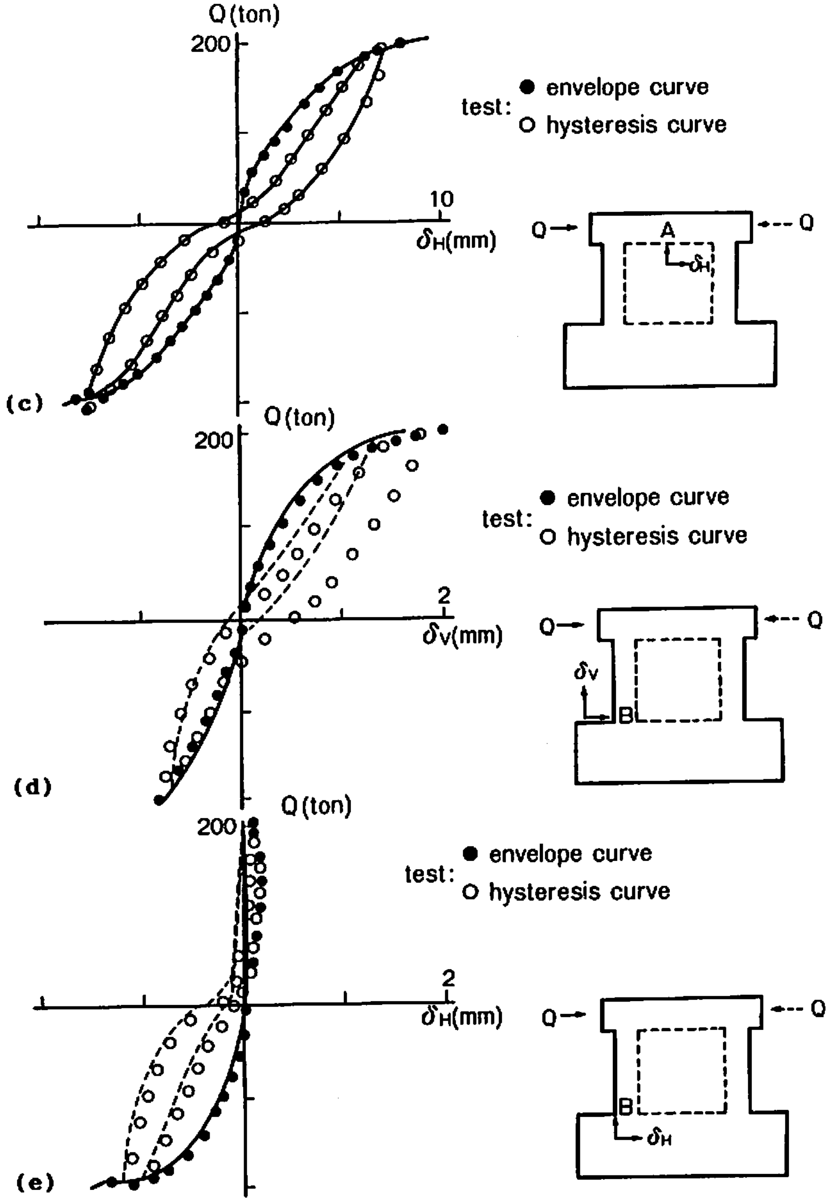

Fig.6(c)(d)(e) FEM analytical results

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Appendix I-References

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