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Analytical Model for RC Panels under Cyclic Load

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Abstract: There are several existing constitutive laws for cracked concrete and steel in RC developed in Okamura laboratory. These models have been developed and verified through reversed uniaxial testing of the RC elements. This study has established the constitutive laws for the RC panel element, combining the existing constitutive laws. The established constitutive has been verified through the test results of the RC panels conducted by Aoyagi et. al., Yoshikawa et al. and Stevens et al. Through verification, the proposed constitutive model has been confirmed to be applicable to the RC panel element subjected to reversed cyclic loading.

Introduction

There are several studies on the constitutive models for cracked concrete and steel in RC such as Okamura·Maekawa model (Okamura), Shima model (Shima), modified Maekawa model (Maekawa), Li·Maekawa model (Li) and Izumo·Okamura model (Izumo). These models have been developed to apply to a RC panel element subjected to reversed cyclic loading.

This study aims at the development of the constitutive law for the RC panel element subjected to reversed cyclic loading, combining the existing constitutive laws.

Premise Conditions

The proposed constitutive law adopts a smeared crack model. Therefore, the constitutive laws are always expressed by the average stresses and the average strains of an objective RC element. The stresses of the RC panel element can be represented by superposition of the

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stresses of concrete and those of steel. The constitutive laws for cracked concrete are always given in the rectangular coordinate system which is composed of the axis crossing the cracked face orthogonally and the axis parallel to the cracked face. The constitutive laws developed by Okamura Laboratory have been used for each concrete stress and steel stress in the RC panel element. It is assumed that steel bars are arranged along two orthogonally crossing directions and their resistance acts only axially in the RC panel element and that the shear resistance of the bars is so small as to be ignored compared with that of concrete.

Constitutive Laws for Tension Stiffening

The tension stiffening under cyclic loading are composed of two parts; the envelope part and the unloading and reloading part (see Fig.1). The former has been proposed by Okamura and Maekawa (Okamura) and the latter by Shima et al. (Shima).

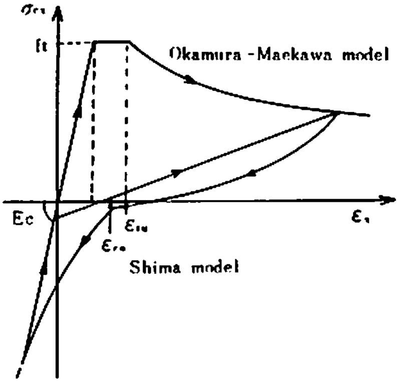


Fig.1 Tension stiffening model for cyclic loading

The tensile stress σ_{cx} for the envelop part is expressed by;

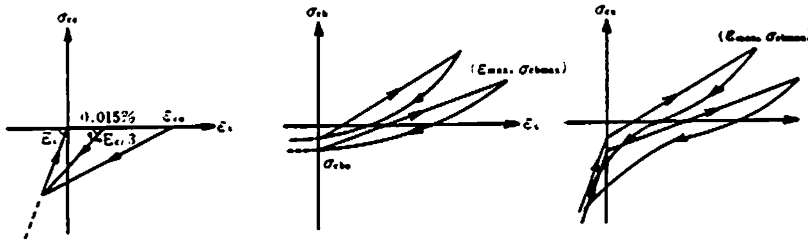
$$\sigma_{cx}=f_t(\varepsilon_{tu}/\varepsilon_x)^c \qquad (1)$$

where f_t indicates the tensile strength of concrete under biaxial stresses, ε_{tu} the strain at the initiation of a crack, ε_x tensile strain, and c the parameter. As the value of the parameter c, 0.4 is be adopted for a deformed bar and 0.2 for a welded steel bar.

For the unloading part and the reloading part, the following equation is used.

$$\sigma_{cx} = \sigma_{cc} + \sigma_{cb} \tag{2}$$

where σ_{cc} indicates the stress transmitted by cracks contacting and σ_{cb} the stress transmitted by bond action.



(a) Concrete stress produced by contact of the crack

(b) Concrete stress produced by the bond action

(c) Tension stiffening for unloading and reloading

Fig.2 Tension stiffening model (Shima)

The criterion for cracks contacting in Shima model is the average tensile strain of 0.015%, which was obtained from the reversed uniaxial loading test (Shima). Under in-plane stress field on which shear deformation will be occurred at the cracked face, the strain ε_{co} at the start of cracks contacting is supposed to be greater than in case of uniaxial stress field. Being considered for possible shear deformation on the cracked face, the tensile strain ε_{co} is expressed by;

$$\varepsilon_{co} = 0.00015 + 0.1 \mid \gamma_{max} \mid \tag{3}$$

where γ_{max} indicates the maximum shear strain that has ever experienced. Therefore, the equation for the contact stress proposed by Shima, should be modified. The equation for the contact stress is represented by a straight line running through two points; the point at the start of cracks contacting and the intersection of the straight line passing the point where the strain is 0.015% and the straight line upon unloading (see Fig.2(a)).

On the other hand, the stress σ_{cb} is represented by a quadratic curve passing the point at the start of unloading ($\varepsilon_{xmax}, \sigma_{cbmax}$) and the vertex ($0, \sigma_{cbo}$) upon unloading. Further, it is represented by a straight line passing both points upon reloading (see Fig.2(b)). The stress at the vertex of the quadratic curve σ_{cbo} is expressed by;

$$\sigma_{cbo}=-0.0016E_c\varepsilon_{xmax} \tag{4}$$

where ε_{xmax} indicates the maximum tensile strain that has ever experienced and E_c initial stiffness of concrete.

Constitutive Law for Compressive Cracked Concrete

It was experimentally proved that the compressive stiffness of concrete in the direction parallel to the cracked face become smaller than that of uncracked concrete(Okamura). Maekawa successfully developed the constitutive law for concrete under compressive stresses for any loading path. The stiffness lowering of cracked concrete is mainly due to stress relaxation in the vicinity of cracks and the authors developed from Maekawa model the following constitutive equation for compressive cracked concrete;

$$\sigma'_{cy}=KE_c(\varepsilon'_{cy}-\varepsilon_p') \tag{5}$$

where K indicates the fracture parameter for cracked concrete and ε'_p the plastic strain obtained from Maekawa model. The fracture parameter K is expressed by $K=\omega K_o$, where K_o indicates the fracture parameter for uncracked concrete. ω is the reduction coefficient and the function of the tensile strain ε_x.

$$\begin{aligned} \omega &=1.0 && \text{for } \varepsilon_x \leqq \varepsilon_1 \\ \omega &=1.0-0.4(\varepsilon_x-\varepsilon_2)/(\varepsilon_1-\varepsilon_2) && \text{for } \varepsilon_1 < \varepsilon_x \leqq \varepsilon_2 \\ \omega &=0.6 && \text{for } \varepsilon_x > \varepsilon_2 \end{aligned} \tag{6}$$

ε_1 is substituted by 0.12% and ε_2 by 0.44%. The reduction coefficient is assumed not to recover upon unloading and reloading. In Maekawa model in which the fracture parameter and plastic strain upon unloading and reloading are assumed not to change, the stiffness of compressive concrete upon unloading and reloading becomes constant. From the experimental observation, it can be seen that the curves has a tendency to get convex toward bottom upon unloading. The convexity in the unloading curve may be attributed to the heterogeneity inside damaged concrete. Lack of such convexity in the unloading curve results in also the lack of convexity in the behavior of the RC panel element. In order to give necessary convexity to the unloading curve, circular arc which has the infinite tangential stiffness at the start of unloading and passes the residual strain point at completion of unloading has been interpolated for the unloading curve. Such analytical model with circular arc can be considered to work well in treating unloading condition, while it is necessary to study further the behavior of concrete under unloading. Fig.3 shows Maekawa model used for the RC panel element.

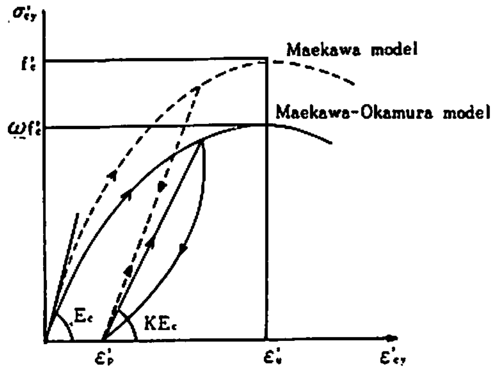


Fig.3 Modified Maekawa model for compressive concrete

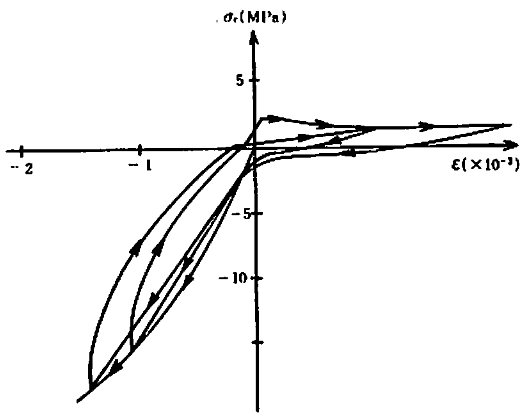


Fig.4 Hysteresis loop of cracked concrete

Constitutive Law for Concrete under Tension-Compression Stress Hysteresis

In case that concrete is exposed to tension-compression stress hysteresis, the constitutive equations for concrete should describe the continuous loop to be relayed from one to another. Fig.4 shows the constitutive law used for cracked concrete under reversed cyclic loading. It is obtained from the above presented models.

Constitutive Law for Shear Deformation along the Crack

Li and Maekawa (Li) developed the constitutive law for the shear behavior along the cracked face by assuming the contact direction and contact area for each contact stress along the cracked face. In their model, the shear stress τ_c along the crack face can be described by eq.(7) for any loading path.

$$\tau_c = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\delta, \omega, \theta) d\theta \qquad (7)$$

where f generally indicates the function among the shear displacement δ in the cracked faces, the crack width ω, and the direction θ along which the contact stress acts. Li·Maekawa model is formulating the shear behavior focusing on the single crack. Li·Maekawa model being used for the smeared crack model, it becomes necessary to settle the average crack spacing. However, there recognised little influence by the average crack spacing in the smeared crack model. The description about that point will be seen in the next chapter where analytical results are described. Li·Maekawa model which doesn't require the average crack spacing is suitable for the analysis of the RC panel element.

Constitutive Law for Steel in the RC Panel Element

A constitutive law for steel in the RC panel element includes the part before yielding of steel, the envelope part after yielding and the unloading part and the reloading part after yielding (see Fig.5). Before yielding, steel is considered elastic. The stiffness of steel itself can be adopted for the stiffness of steel in the RC panel element.

For the constitutive law for the envelope part, the Izumo and Okamura model (Izumo) that was derived from the assumption that steel stress distribution after yielding could be expressed by a cosine function has been adopted. For the unloading and the reloading part, Shima model (Shima) which is not only easy to handle but also can give accurate results has been adopted.

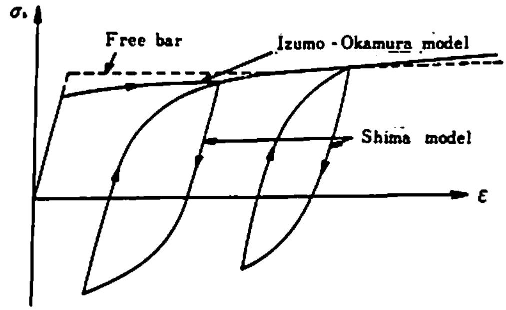


Fig.5 Constitutive model for steel under cyclic loading

Analytical Method

The authors have developed the numerical programing of the constitutive laws for the RC panel element. The established constitutive law results in the form of a three-element non-linear simultaneous equations and then the analysis has been done by solving them to get the values of strains with the stresses acting on the RC panel element. The non-linear simultaneous equations have been solved using Newton-Raphson method. The value of the stiffness in the analysis uses the constant stiffness that is always greater than the tangential stiffness. This is due to take into consideration of the possibility that the sudden change of stiffness in the hysteresis curve, would make vibrate the solutions. Negative stiffness has not been used because of the negative stiffness making the corresponding stiffness matrix peculiar. Therefore, in this analysis, the negative stiffness value has been substituted for zero.

Analytical Conditions for the RC Panel Element

The proposed analytical model can be verified by the test results by Aoyagi et al. (Aoyagi), Yoshikawa et al. (Yoshikawa) and Stevens et al. (Stevens), which specimens were subjected to cyclic or reversed cyclic in-plane stresses. In this analysis, the tensile strength of concrete f_t shall be modified in a way that the tensile strength may coincide with the cracking load of the test result through trials and errors. This is the reason the behavior of the RC panel element after cracking cannot be adequately traced, if the estimation of cracking load is different from the testing results. For easy comparison of data between the estimated by analysis and the observed, a few loops from the hysteresis curve of the test results are used.

Analytical Results and Remarks

Fig.6 shows the analytical results and the observed test results. As for the specimen No.11 (Aoyagi) with isotropic arrangement of bars, it is subjected to cyclic uniaxial tensile stress. The specimen No.11 is much effected by tension stiffening. The specimen SP1 (Yoshikawa) having the isotropic arrangement of bars, it is subjected to reversed cyclic shear stress. In this specimen, steel will be yielded. Therefore, the specimen SP1 is effected by the constitutive model for tension stiffening, compressive concrete and steel after yielding.

As for the specimen SE10 (Stevens), where reversed cyclic shear and compressive stresses act on, shear stress will act on the cracked face due to the anisotropic arrangement of bars. Before analyzing the specimen, calculation giving two extremely different values 5cm and 100cm as the average crack spacing was done. However, the results were completely same in both cases and Li·Maekawa model were confirmed to work well also in analyzing the RC panel element regardless of the crack spacing. The analytical results using the proposed model seems to have good correspondence with the test results.

Conclusions

The constitutive laws for the RC panels has been developed, using the existing constitutive laws for cracked concrete and steel in RC. Further, the established constitutive model has been verified through comparison with test data of the RC panels. As far as verification has been done, the proposed model has been proved effective for the analysis of the RC panel element subjected to reversed cyclic loading.

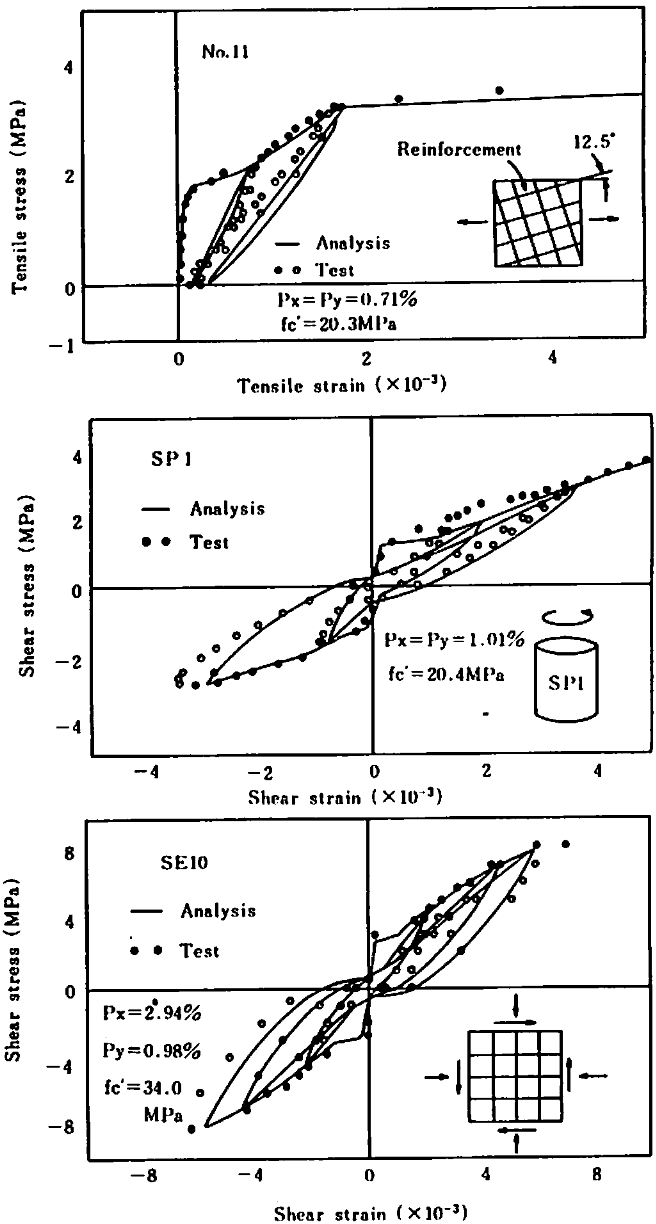


Fig.6 Analytical results of the RC panels

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