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# AVERAGE STRESS-STRAIN RELATIONSHIP IN POST YIELD RANGE OF STEEL BAR IN CONCRETE

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SYNOPSIS

Uniaxial tensile tests were conducted on prismatic reinforced concrete specimens. Steel stress distributions along a bar in addition to elongations of each specimen were measured and the average stress-strain relationship of a steel bar in post-yield range was obtained. The average stress of a bar in concrete starts to yield below the yield point of the bare bar and the yield plateau disappears. The average stress-strain relationship in post-yield range can be predicted by assuming a cosine curve for the stress distribution along the bar.

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## 1. INTRODUCTION

When a reinforced concrete structure is subjected to severe forces such as ground motions of strong intensity, displacements well in excess of the yielding may be induced. The tension stiffness of concrete plays a significant role on the deformational behavior of the reinforced concrete structure in post-yield range of steel (1). However, researches concerning the tension stiffness of a reinforced concrete member have dealt with only in elastic range but not in the post-yield range.

The tension stiffness of a reinforced concrete element is usually obtained by superimposing the average stiffness of concrete and that of steel bars, and the stiffness of the bars in concrete is usually modeled as same as that of bare bars. However, this modeling is not applicable for the post-yield range of steel because the existence of bond between bars and concrete after yielding of steel causes the average stress-strain relationship of the bars in concrete to be distinct from that of the bare bars. The object of this research is to obtain the average stress-strain relationship in the post-yield range of a bar in concrete.

Uniaxial tensile tests were carried out using prismatic reinforced concrete specimens. Before yielding of steel, the average stress of a bar in concrete can be directly determined from the average strain of the specimen by introducing the constitutive laws of the steel. However, after the bar starts to yield at a cracked section in a reinforced concrete element, the yielding at the cracked section does not extend into uncracked one between cracks so quickly due to the existence of bond (2), and both yielded and un-yielded portions exist along the bar. This makes difficult to determine the average steel stress directly from the average steel strain in the post-yield range. Therefore, the appropriate method to determine the average steel stress from the average steel strain has to be established. The stress distribution of a bar along the specimen was measured to obtain the average steel stress. The relationship between the average stress and the average strain of a bar was predicted by modeling the stress distribution curve of the bar.

## 2. EXPERIMENTS

### 2.1 Experimental Conditions

The important parameters for the tension stiffness of a reinforced concrete element are concrete strength, reinforcement ratio, yield strength of steel and curing conditions. Combining these factors as shown in Table 1, six specimens were tested.

Drying shrinkage causes tensile stress in concrete because the shrinkage of concrete is restrained by the reinforcing bar. Therefore, the apparent tensile strength of concrete decreases and the cracking resistance is reduced. The specimen No.1 was cured in water to prevent the drying shrinkage and was compared with the specimen No.2 cured in air.

The specimens No.2 and No.3 were used to investigate the influence of concrete strength on the tension stiffness. The specimens No.3 and No.4 were used to investigate the influence of reinforcement ratio.

Since the average stress-strain relationship of a reinforced concrete element is considered to vary with the stress-strain relationship of steel, the steel bars having different yield strength were used in specimens No.4, No.5 and No.6.

Table 1 Outline of test specimens.

| Specimen No. | f'c (MPa) | p (%) | Steel Bar | Curing condition |
|---|---|---|---|---|
| 1 | 45 | 1.0 | SD50 | in Water |
| 2 | | | | in Air |
| 3 | 25 | | | |
| 4 | | 0.6 | | |
| 5 | | | SD70 | |
| 6 | | | SD30 | |

f'c: Compressive strength of concrete
p : Reinforcement ratio

## 2.2 Specimens

The dimensions of the specimens are shown in Fig.1. A steel bar was arranged in the center of the concrete prism. A deformed bar having a diameter of 19mm was used and reinforcement ratio was adjusted by changing the cross-sectional area of concrete. When we investigate the average strain and stress of the steel bar in a cracked reinforced concrete member, the longer specimen which includes more cracks will give higher accuracy. The length of specimen was determined to be as long as possible under the experimental limitations.

The bar was fixed centrically along the form with a special attention to arrange the bar straightly. Concrete was cast in the vertical direction perpendicular to the bar. The specimen No.1 was cured in water from just after removing the form till the day before the test to prevent the drying shrinkage. Furthermore, lacquer paint was applied to the surface so that the drying could be avoided. The other specimens was cured by being covered with the wet cloth during a few days after casting of concrete and cured in air after that.

Strains of the bar were measured and converted to stresses by using the stress-strain relationship of the bare bar. In order to measure the steel strain distribution along the bar, foil resistance strain gauges having length of 5mm were attached on the opposite faces of the bar at an interval of 10 ribs. The reason why the gauge interval was controlled by the number of ribs along the bar is to obtain the consistent cross-sectional area of the locations where the strain gauges were attached to.

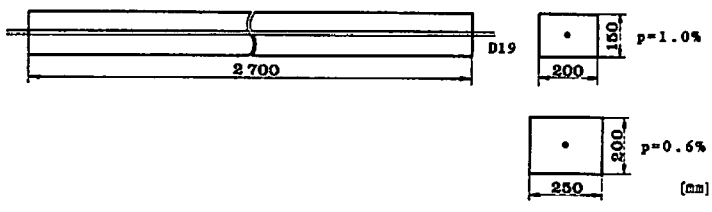



Fig.1 Specimens.

## 2.3 Steel bars

The measurement of steel strain along the length of the bar is the most important work in the experiment. If an ordinary steel bar was used, ribs of the bar should have been removed to attach the strain gauges resulting in reduction of the cross-sectional area of the bar. To solve this problem, screw-shaped deformed bars without longitudinal ribs were used.

The properties of bars are given in Table 2. The cross-sectional area used in the analysis was determined by dividing the measured volume obtained from submerged weight by its length. The measured stress-strain curves of the bars are plotted in Fig.2. The solid lines in the figure were used in the analysis.

Table 2 Properties of steel bars.

| Steel bar | SD30 | SD50 | SD70 |
|---|---|---|---|
| Diameter for analysis, mm | 19.5 | 19.5 | 19.5 |
| Young's modulus, GPa | 190 | 190 | 190 |
| Yielding point, MPa | 350 | 610 | 820 |
| Initial strain of strain hardening, % | 1.65 | 1.40 | 0.60 |
| Stiffness at the strain hardening range, GPa | 3.7 | 5.9 | 2.8 |

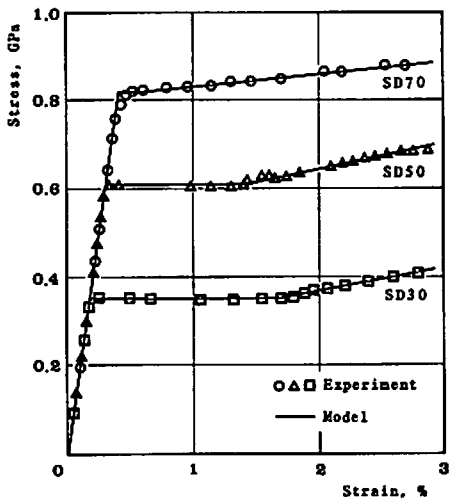



Fig.2 Stress-strain relationships of steel bars.

## 2.4 Test method

The testing apparatus was set horizontally on the floor as shown in Fig.3. One end of the steel bar protruding from the end of the specimen was fixed to the prestressed concrete frame and the tensile load was applied to another end of the bar by an center hole oil jack. The friction between the specimen and the floor was prevented by setting steel rollers under the specimen.

In addition to the strains and the applied load, the elongations of the specimen were measured. The elongation was obtained from the relative displacement of the bar at the both end of the specimen. The stainless wires having diameter of 0.3mm had been adhered to the opposite faces of the bar at the end faces of the concrete with solder before the casting. The wires were pulled out of the

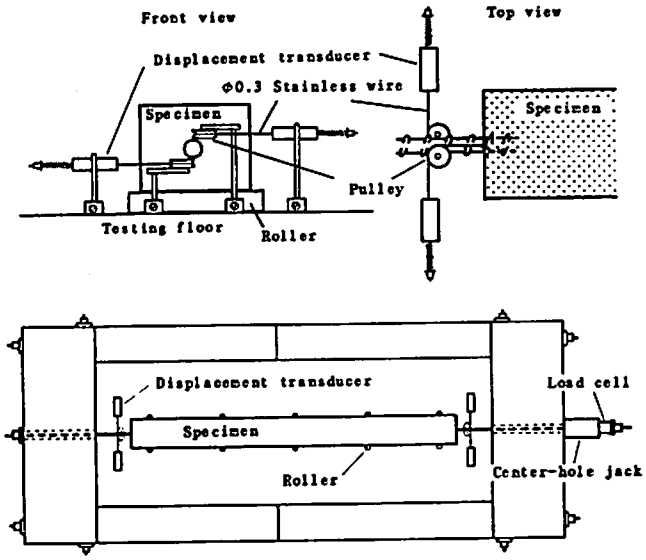



Fig.3 Testing apparatus.

specimen and connected to electrical displacement meters through pulleys as shown in the upper part of Fig.3. The pulleys and displacement meters were fixed on the floor. The values of the elongation and strain were derived from the average of two values measured on the opposite faces of the bar.

## 3. EXPERIMENTAL RESULTS AND DISCUSSIONS

### 3.1 Strain and Stress Distributions

The typical strain and stress distributions in post-yield range of the steel bar along the length of the specimen are shown in Fig.4. The method to determine the stress distribution and the average stress along the bar is as follows.

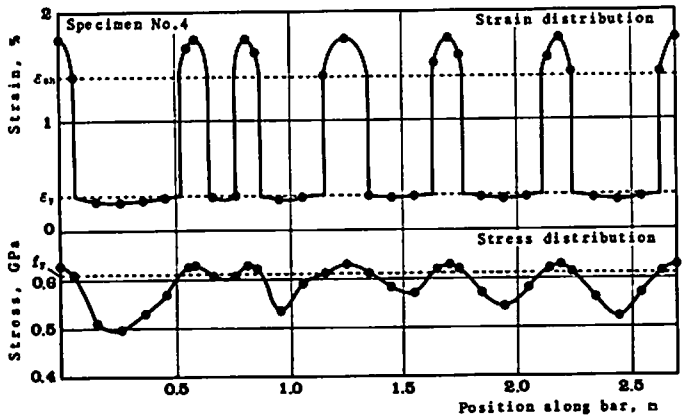


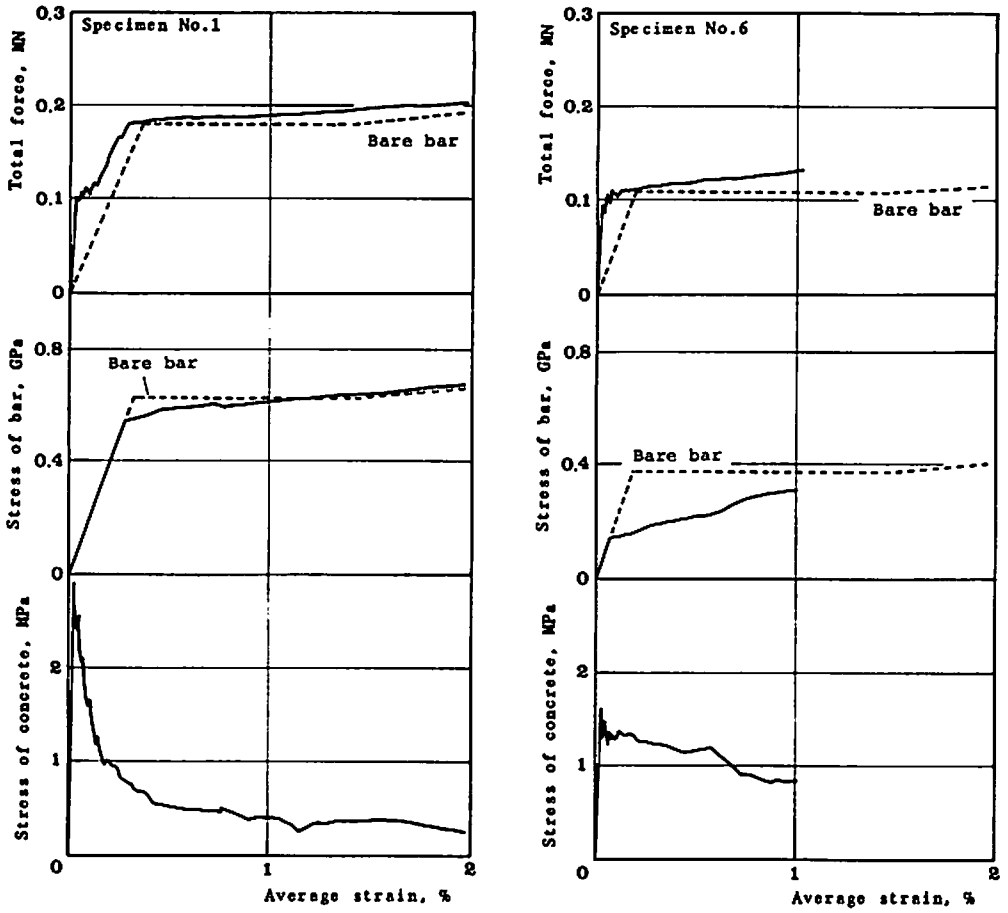

Fig.4 An example for distributions of strain and stress in post-yield range.

Firstly, the steel stresses at the measured points were decided from the strain measurement using the stress-strain relationship shown in Fig.2. Secondly, the stress distribution was represented by 2nd order polynomial curve which can fit three neighboring points considering that the stress distribution can be expressed by a smooth curve even after the yielding. The average stress was obtained by dividing the integration of the stress distribution curve by length of the specimen.

Strains are in elastic range where the stress distribution curves are smaller than the yield strength and the strains are in strain hardening range where the stress distribution curves are larger than the yield strength. The steel strain jumps up to the initial strain of strain hardening when the stress equals to the yield strength (2).

### 3.2 Average Stress-Strain Relationships

The load-average strain and the average stress-average strain relationships of specimens No.1 and No.6 are shown in Fig.5. The average strain was obtained




(a) Specimen No.1 (b) Specimen No.6

Fig.5 Average stress-strain relationships.

from the measured elongation of the specimen. The load carried by the steel bar is calculated by multiplying the average stress of the bar by its cross-sectional area. The average concrete stress is obtained by subtracting the load carried by the steel bar from the applied load and dividing the subtraction result by the sectional area of concrete.

By means of the bond stress between the bar and concrete, the concrete between cracks carries tensile force even in the post-cracking range. As a result, the tensile stiffness of a reinforced concrete element is higher than that of a bare bar alone. It reveals from Fig.5 that concrete can carry the tensile force even after the bar has yielded. Furthermore, there is no tendency of sudden reduction of tension stiffening effect in the post-yield range. In the specimen No.1, for example, it can be observed from Fig.5(a) that concrete still can resist the load equivalent to roughly 10% of its tensile strength when the average strain approaches 2%. This results in higher post-yield stiffness of the reinforced concrete element than that of a bare bar alone.

The average stress-strain curve of the steel bar in a reinforced concrete element has the following distinction from a bare bar.

1) The yield point is lower.

2) No yield plateau exists.

The relationship between the steel stress at cracked sections and the average stress of the bar and concrete is expressed by

$$\sigma so = \sigma s + \frac{\sigma c}{p} \qquad (1)$$

where σso: steel stress at cracked sections
σs : average steel stress
σc : average concrete stress
p : reinforcement ratio (=As/Ac), As: steel area, Ac: concrete area.

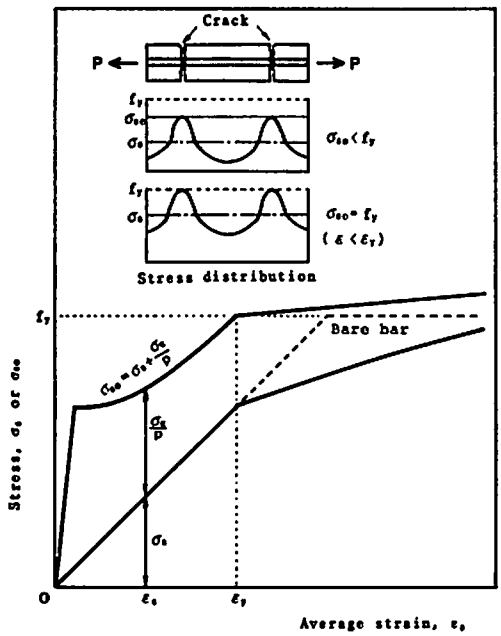



Fig.6 Average stress and average strain of a steel bar when yielding occurs.

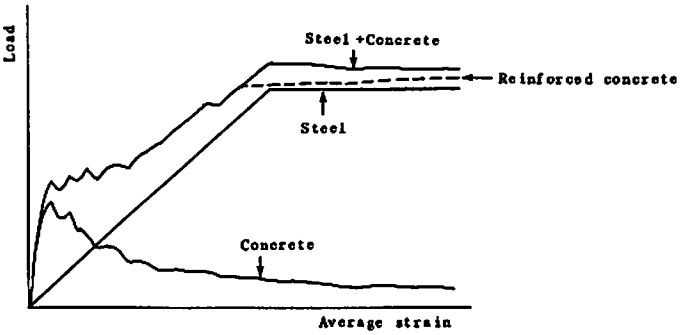



Fig.7 Fault in superimposing stress of steel and concrete.

The steel stress distribution between cracks and the steel stress-average strain diagram are illustrated in Fig.6. When a bar starts to yield at cracked sections, the steel stress between cracks should be less than the yield stress. This causes the average stress-strain relationship of steel to have lower yielding point than that of a bare bar. Therefore, to evaluate the tensile stiffness of a reinforced concrete element, it is irrational to superimpose the average stress-strain curve of concrete on the stress-strain curve of the bare bar because it may result in higher yielding point than the actual point in the reinforced concrete element as shown in Fig.7.

It can be recognized from Eq.(1) and Fig.6 that the lower the reinforcement ratio, the lower the yield strength of steel or the higher the concrete strength is, the lower the yielding point of the average stress-strain curve of the steel bar in a reinforced concrete element will be. The yielding point of the average stress-strain curve of the steel bar in specimen No.6 is considerably lower than that of the bare bar as shown in Fig.5(b). In this specimen, the yielding of steel started just after the cracking had occurred because the specimen contained very low reinforcement ratio with mild yield strength. The height of yielding point in the average stress-strain curve is lower than half of that of the bare bar.

If there was no strain hardening in steel, the steel bar at cracked sections failed under the yielding load without any elongation of the specimen. Due to the strain hardening, the steel bar at cracked sections can carry more load, and the strain jumps up to the strain hardening range immediately after the steel bar yields in concrete as shown in Fig.4. Due to bond, differences in stress at both sides of an infinitesimal length at yielding location of the bar in concrete can be produced even in the post-yield range (2). This causes no yield plateau in the average stress-strain curve of steel in a reinforced concrete element.

After the strain of steel has entered its strain hardening range at cracked sections, the yielding penetrates into the uncracked part gradually as long as the applied load increases. When the strain becomes large enough, the average stress-strain curve approaches that of the bare bar. This is because the larger the steel strain is, the lower the bond stress and the smaller the influence of tension stiffening effect are. As a consequence, the earlier the yielding of the average stress-strain curve is, the higher becomes the gradient of the average stress-strain curve after yielding.

## 4. ANALYTICAL APPROACH

### 4.1 Modeling the Stress Distribution Curve

The tensile stiffness of a reinforced concrete element can be expressed by the sum of the stiffness of steel bars and that of concrete. The tensile stiffness in the post-yield range can be predicted if the average stress-strain relationship of steel bars and that of concrete in the post-yield range are known.

The average stress of a bar can be obtained from the stress distribution along the bar and the average strain can also be obtained from the strain distribution. As one of the either distribution is given, another one can be obtained by using the stress-strain relationship of the bare steel. The average stress-strain relationship can be predicted by assuming the stress or strain distributions along the bar.

There are three ways of approaches to obtain the stress or strain distributions.
1) assuming a bond stress-slip relationship (3)
2) assuming a bond stress distribution (4) and
3) assuming a stress distribution or strain distribution.

If the bond stress-slip relationship is assumed, it will be unique at every section along the bar. However the bond stress is zero at the cracked sections and the longitudinal cracks reduces the bonding ability (5). Therefore, it is impossible to assume a unique bond stress-slip relationship near the cracked sections. As the method to treat the bond reduction near the cracked sections, the method using different bond stress-slip relationships with the location along the bar (6) or the method in which the bond stress is reduced by multiplying a reduction factor to the bond stress obtained from the bond stress-slip relationship (7) were proposed. However it is more practical to assume the bond stress distribution or the steel stress and strain distributions than to assume the bond stress-slip relationship in case of predicting the tension stiffness. Although the bond stress distribution can be expressed by a continuous smooth curve in the elastic range of steel, it is difficult to express the bond stress distribution in the post-yield range by a simple form because of the sudden reduction of the bond stress due to the yielding of the steel (2). Therefore, in predicting the average stress-strain relationship in the post-yield range, it is better to assume the stress distribution or the strain distribution.

From the distributions of stress in post-yield range shown in Fig.4, the stress distribution is considered to be expressed by a continuous smooth curve even in the post-yield range. Therefore, the authors assume the stress distribution curve in order to predict the tension stiffness in the post-yield range.

The average strain is mostly controlled by the length of the region where the bar has yielded. Therefore, the stress distribution model should give the accurate length of the yielded region. The region can be determined as the range from the cracked section to the intersection of the stress distribution curve and the yield stress. Therefore, the stress distribution curve should be the one to give the accurate intersection point of the yield stress.

The crack spacings observed in the experiment were not the same in a specimen so that the stress distribution curves in each segment between cracks were not the same. The assumed stress distribution curve should express, not the stress distributions in each segment, but the average stress distribution curve in a specimen. This is because the assumed stress distribution curve is used to predict the average values of the stress and strain.

The bond stresses at cracked sections and at the center of a segment are zero and the bond stress is expressed as a differential function of the stress distribution curve. Therefore, the stress distribution curve should have a form such that the slopes of the tangent at the cracked sections and the center of segment are zero. The cosine is one of the functions which agree with the above conditions. The comparison of the cosine curves with the experimental results of the stress distribution in each specimen are shown in Fig.8 to Fig.10. The experimental results are represented by taking the average of stress distributions in halves of each segment. The cosine curves are determined by adjusting the maximum stress and average stress of the curves to those of the experimental results.

The influence of the difference of yield strength of the steel bar, the difference of concrete strength and reinforcement ratio, and the difference of the curing condition on the steel stress distribution curve are demonstrated in Fig.8, Fig.9 and Fig.10, respectively. These figures indicate that the stress distribution can be represented by cosine curve independent on the yield strength of the bar, concrete strength and on crack spacings which are varied with drying shrinkage or reinforcement ratio. This fact shows that the steel stress distribution along the bar in post-yield range can be assumed by a cosine curve in the analysis of reinforced concrete element level.

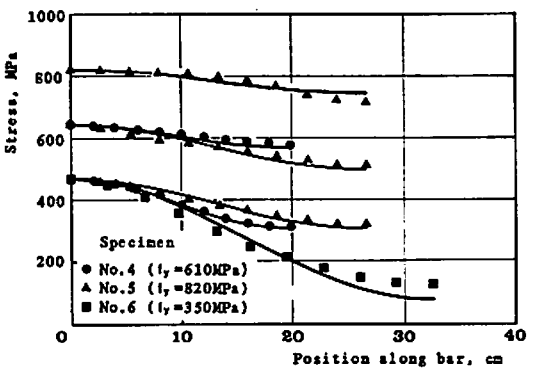



Fig.8 Comparison of cosine curves with experimental results of steel stress (effect of yield strength of steel bar).

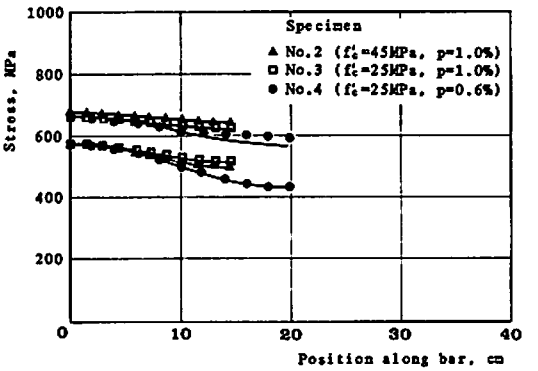



Fig.9 Comparison of cosine curves with experimental results of steel stress (effect of concrete strength and reinforcement ratio).

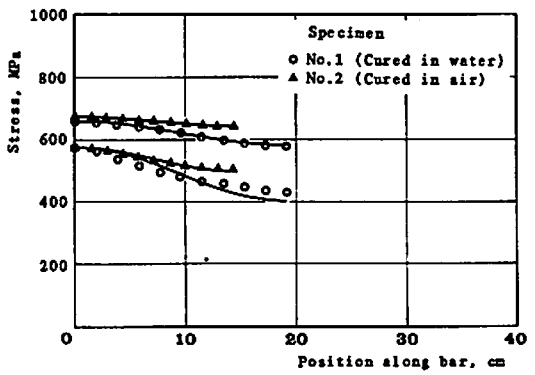



Fig.10
Comparison of cosine curves with experimental results of steel stress (effect of drying shrinkage of concrete).

When the stress distribution of steel is assumed by a cosine curve, the average stress lies in the middle between the maximum stress and the minimum stress. In the case that the average stress is lower than the half of the maximum stress, the minimum stress must be assumed to be negative value. As compressive stress never caused in a specimen under monotonic tensile loading, the assumption of a cosine curve for the stress distribution is restricted in its application. This situation occurs when the crack spacing is very large. The cracking resistance for low reinforcement ratio nearly equal to Acft and the yielding load are expressed by Asfy, where ft is tensile strength of concrete and fy is yield strength of steel. When the ratio of these value p fy/ft is close to 1, it is possible that the stress distribution can not be expressed by a cosine curve.

## 4.2 Calculating Procedure

The calculating procedure to obtain the average stress-strain relationship of a steel bar after yielding by assuming a cosine curve for the stress distribution curve is described as follows.

(1) An average stress-strain relationship for concrete in tension should be given. This relationship must be able to consider the tension stiffening effect both before and after the yielding of steel.

(2) The yielding in the average stress-strain curve occurs when the steel stress at a cracked section reaches the yield strength fy as given by

$$\sigma so = \sigma s + \frac{\sigma c}{p} = fy \tag{2}$$

After yielding,
(3) An average strain $\varepsilon$ is given.
(4) The corresponding average stress of concrete $\sigma c$ is calculated by using the average stress-strain relationship of concrete.
(5) The amplitude of steel stress distribution curve "a" is determined by the difference between the maximum stress at cracked sections $\sigma so$ and the average steel stress $\sigma s$ as follows.

$$a = \sigma so - \sigma s \tag{3}$$

(6) An average steel stress $\sigma s$ is assumed.
(7) The steel stress distribution curve is given by

$$\sigma s(x) = \sigma s + a \cos x \quad ( 0 \leqq x \leqq \pi ) \tag{4}$$

(8) The corresponding steel strain distribution $\varepsilon(x)$ is obtained.
(9) The average strain is calculated from the steel strain distribution as given by

$$\varepsilon = \frac{1}{\pi} \int_0^{\pi} \varepsilon s(x)\, dx \tag{5}$$

(10) Unless the given average strain is considered equal to the calculated strain within a reasonable limit, iterative calculation from (6) to (9) is done.
(11) The average steel stress corresponding to the given average strain is decided when the calculated average strain is considered equal to the given one.

## 4.3 Verification of the Model

The accuracy of the model can be verified by comparing the analytical load-strain curve with the experiments. The experimental results can be used for the average stress-strain relationship of concrete in the analysis.

The analytical load-strain curve of a reinforced concrete uniaxial tension

member is obtained by superimposing the average stress-strain relationship of the steel and that of concrete. The load-strain curves obtained from the experiments and from the analysis are shown in Fig.11. The solid lines show the analytical results in which the experimental results of average stress-strain relationship of concrete as shown in the bottom part of Fig.5 was used. As shown in Fig.11, the analytical results well coincides with the experimental ones. This indicates that the modeling of the steel stress distribution by a cosine curve is appropriate.

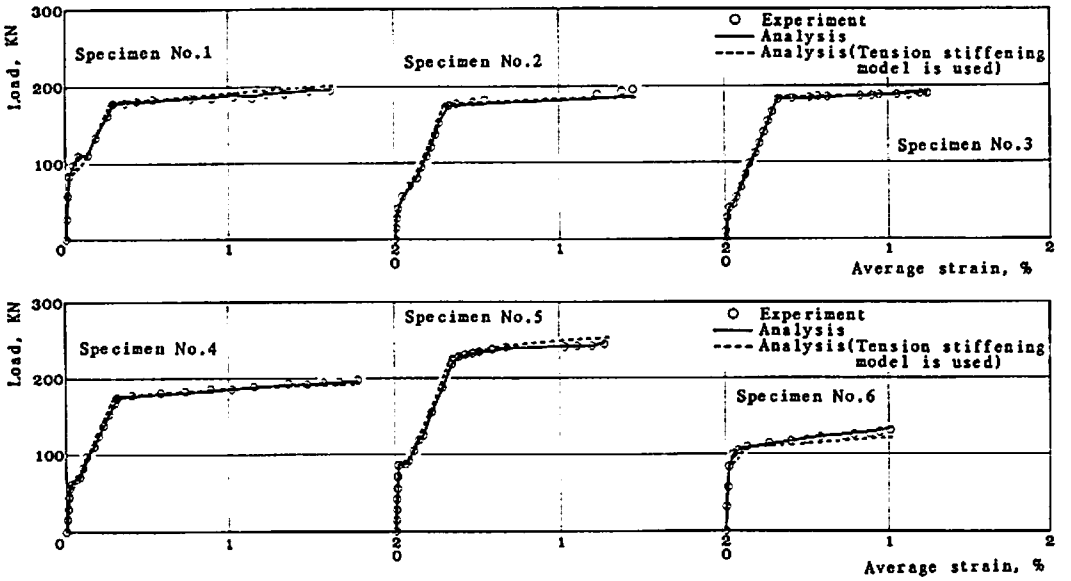



Fig.11 Comparison of analytical and experimental results of load-average strain curves.

In the analysis of reinforced concrete structures or elements, the average stress-strain relationship of concrete in tension shall be given. For example, the average stress-strain model of concrete introduced by Okamura et al.(1) were demonstrated. Their model is given by

$$\sigma c = ft\left(\frac{\varepsilon tu}{\varepsilon}\right)^{c} \qquad (6)$$

where ft : tensile strength of concrete
εtu : cracking strain equal to 0.02%
c : a constant depending on bond characteristic, 0.4 for deformed bars.

The experimental results were used for the tensile strength of concrete because the tensile strength of concrete has important influence on not only the cracking load but also the behavior even after the cracking. The analytical results of the load-strain relationships obtained by using Eq.(6) are illustrated by the broken lines in Fig.11. The analytical results agree well with the experimental results. However, a little difference can be seen in some cases. It is indicated that the accuracy of the average stress-strain model of concrete affects the accuracy of prediction for the tension stiffness in post-yield range. Therefore, it is necessary to use an accurate model of concrete in order to predict the tension stiffness with high accuracy.

In case of the specimen No.6, the minimum stress whose stress distribution curve was represented by a cosine curve became negative at the time just after

yielding of the bar because the yielding started when the average steel stress was lower than the half of the yield stress as shown in Fig.5(b). To solve this problem, the average steel stress was obtained from the range in positive stress excepting the range of negative stress. The average strain is almost controlled by the plastic length (2), so that the modification of stress distribution curve in a part of elastic range does not affect the average strain.

## 5. CONCLUSIONS

(1) As the concrete between cracks carries tensile force even after yielding of steel, the tensile stiffness of a reinforced concrete element is higher than that of the bare bar even in post-yield range.

(2) The yield point of the average stress-strain curve of the steel bar in a reinforced concrete element is lower than that of the bare bar and no yield plateau exists in the curve.

(3) In the post-yield range, the average stress of a steel bar is obtained from the average strain by modeling the steel stress distribution along the length of the bar. A cosine curve is considered as one of the suitable stress distribution function.

(4) A method for calculating the tensile stiffness of an uniaxial tension member in post-yield range of steel is proposed.

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