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CONSTITUTIVE EQUATIONS FOR CONCRETE UNDER PLANE STRESS CONDITIONS

Koichi MAEKAWA
Assistant Lecturer of Civil Engineering
Technological University of Nagaoka
Niigata, Japan

Hajime OKAMURA
Professor of Civil Engineering
University of Tokyo
Tokyo, Japan

1. Equivalent Stress - Strain Relationship

In case of concrete under plane stress conditions, there exists an elastic zone on a stress space or a strain space, where the stress or the strain is very small. When strained beyond the boundary of the elastic zone, additional irreversible deformation occurs and the elastic zone expands correspondingly (See Fig.1.). A subsequent elastic zone is developed even in the strain softening conditions. Within this newly developed elastic zone, we found the linear relationship between the equivalent stress S and the equivalent elastic strain Ee as

$$S = Eo\ K\ Ee \qquad (1)$$

where Eo is a constant corresponding to the initial stiffness and K is the ratio of linear elastic stiffness to the initial one. The equivalent stress and the equivalent elastic strain is defined as

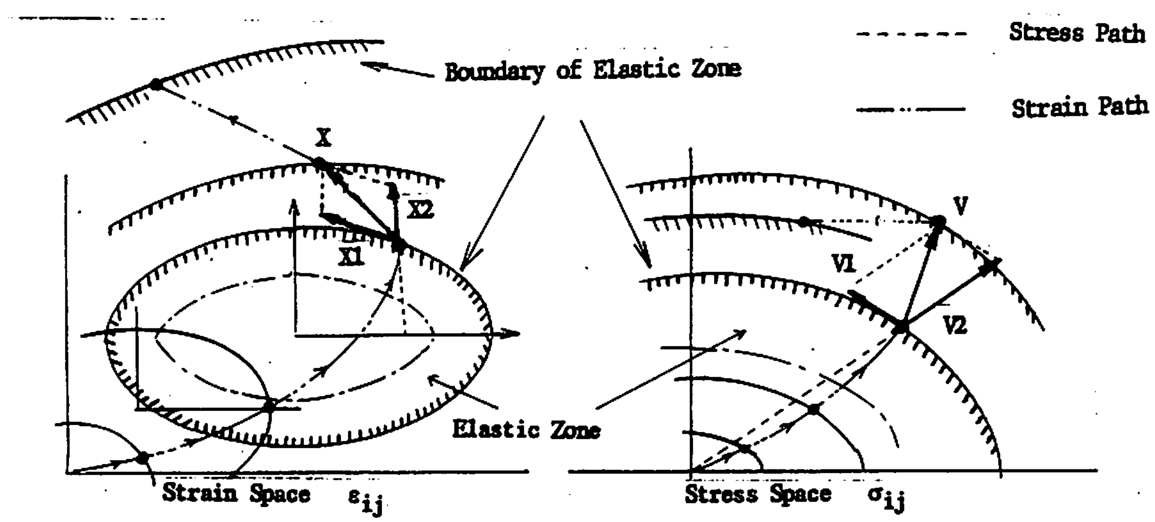

Fig. 1 Elastic Zones on Stress and Strain Spaces

$$S = \sqrt{\left(\frac{0.60\,\sigma_o}{fc}\right)^2 + \left(\frac{1.30\,\tau_o}{fc}\right)^2}$$

$$\sigma_o = \sqrt{2}\,\frac{\sigma_{xx} + \sigma_{yy}}{2}$$

$$\tau_o = \sqrt{2\left(\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2\right)} \qquad (2)$$

$$Ee = \sqrt{\left(\frac{0.62\,\varepsilon_o}{\varepsilon_{oo}}\right)^2 + \left(\frac{0.98\,\gamma_o}{\varepsilon_{oo}}\right)^2}$$

$$\varepsilon_o = \sqrt{2}\,\frac{\varepsilon_{exx} + \varepsilon_{eyy}}{2}$$

$$\gamma_o = \sqrt{2\left(\left(\frac{\varepsilon_{exx} - \varepsilon_{eyy}}{2}\right)^2 + \varepsilon_{exy}^2\right)} \qquad (3)$$

where fc is the uniaxial compressive strain and εoo is the corresponding compressive strain.

Then the boundary should be defined as Ee equal to Eemax and the corresponding equivalent stress Smax shall be

$$Smax = Eo\ K\ Eemax \qquad (4)$$

In the plastic theory, K is considered constant and the relation between the both boundaries on the stress and strain spaces is unchanged. However, in case of concrete, K is not constant and from the experimental results of biaxial unloading paths, we have found the K values shall be calculated by

$$K = \exp(-0.73Emax(1-\exp(-1.25Emax)))$$

$$E = \int \frac{\partial Ee}{\partial \varepsilon_{eij}}\, d\varepsilon_{ij} \qquad (5)$$

as shown in Fig.2, where E is a strain path-dependent index E called as the total equivalent strain. This nonlinearity is because the local buckling due to the extension of microcrackings and disappearance of some volume in which the strain energy is reserved. The parameter K expresses this behavior and accordingly we call this parameter as a fracture parameter.

When the subsequent elastic zone is expanded, it also moves on the strain space according to the kinematic shift of plastic strains. The effective plastic strain defined as

$$\varepsilon_{pls} = \int \sqrt{d\varepsilon_{pij}\ d\varepsilon_{pij}} \qquad (6)$$

cannot be applied for concrete under compression - tension stress state as shown in Fig.3. We have newly introduced the equivalent plastic strain Ep representing the shift of the elastic zone on the strain space. We formulated Ep as the function of the total equivalent strain

$$Ep = \int \frac{\partial Ee}{\partial \varepsilon_{eij}}\, d\varepsilon_{pij} \qquad E = Ee + Ep$$

$$Ep = Emax - \frac{20}{7}(1-\exp(-0.35Emax)) \qquad (7)$$

based on the test results shown in Fig.2.

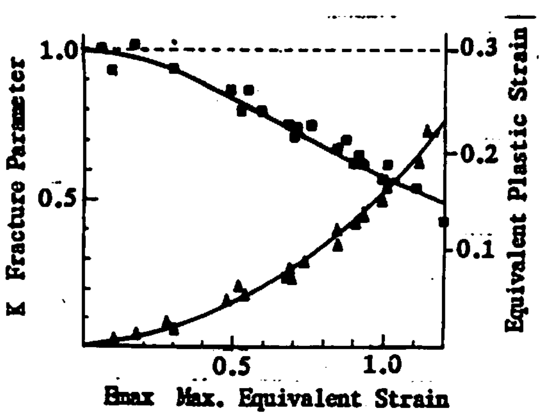

Fig.2 Fracture Parameter and Equivalent Plastic Strain

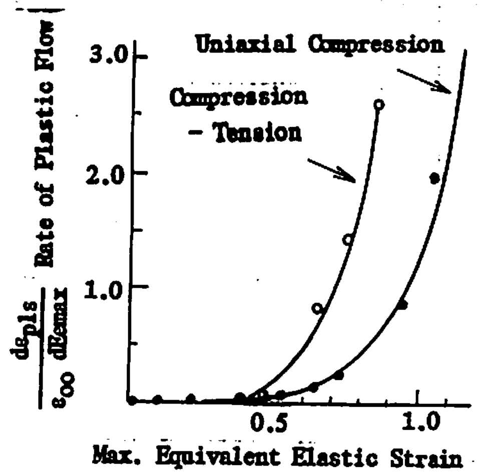

Fig.3 Effective Strain in the Plastic Theory

2. Flow Rule

In completing the system of constitutive equations, it is required to formulate the directional correlation between the stress and strain invariant vectors and the principal stress direction under an arbitrary strain path. Within the elastic zone, the directional relationship can be assumed isotropic and symmetric as

$$\begin{bmatrix} s_o \\ \gamma_o \end{bmatrix} = \frac{1}{E^*} \begin{bmatrix} 1-\nu & 0 \\ 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_o \\ \tau_o \end{bmatrix} \qquad (8)$$

E^* : Proprotional coefficient

The Poisson's ratio was experimentally obtained as shown in Fig.4. When the strain moves within the elastic zone, two stress invariants can be obtained by solving Eq.1 and Eq.8 simultaneously.

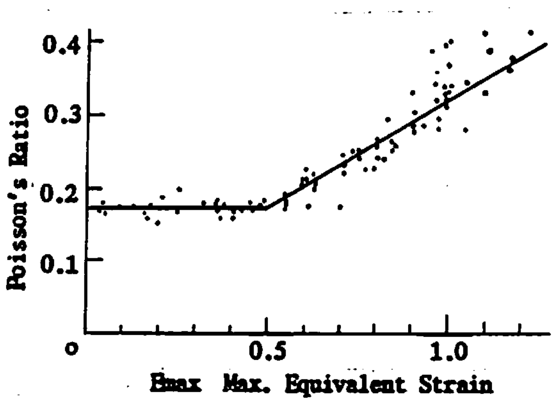

Fig.4 Poisson's Ratio in Elastic Zone

To formulate the flow rule governing the direction of the plastic rate vector, we introduced the directional correlaton between stress invariant rate and total strain invariant rate. The stress invariant rate can be divided into two components, V1 and V2 as

$$V = \begin{bmatrix} d\sigma_o \\ d\tau_o \end{bmatrix} = V1 + V2 = \begin{bmatrix} -\dfrac{\partial S}{\partial \tau_o} \\ \dfrac{\partial S}{\partial \sigma_o} \end{bmatrix} dj + \begin{bmatrix} \sigma_o \\ \tau_o \end{bmatrix} dl \tag{9}$$

where the first term V1 is the tangential component to the stress surface of constant S and the second term V2 is the parallel component to the stress invariant vector. The total strain invariant rate has two components X1 and X2, corresponding to V1 and V2 respectively as shown in Fig.1.

The direction of X1 must coincide with the tangential vector on the boundary of strain space due to the continuity condition. The direction of X2 was obtained from the experiments including non-proportional loading paths as shown in Fig.5 which indicates all the component X2 of the same strain surface converge to the same point. Accordingly, we get the total strain invariant rate X expressed by

$$X = \begin{bmatrix} \dfrac{\partial \varepsilon_o}{\partial \varepsilon_{eij}} d\varepsilon_{ij} \\ \dfrac{\partial \gamma_o}{\partial \varepsilon_{eij}} d\varepsilon_{ij} \end{bmatrix} = X1 + X2 = \begin{bmatrix} -\dfrac{\partial Ee}{\partial \gamma_o} \\ \dfrac{\partial Ee}{\partial \varepsilon_o} \end{bmatrix} dk + \begin{bmatrix} \varepsilon_o - \alpha(Ee) \\ \gamma_o - \beta(Ee) \end{bmatrix} dm \tag{10}$$

The flow direction of plastic strain invariants is now obtained from Eq.8, 9 and 10. Typical cases are shown in Fig.6. For low stress level, the direction is approximately parallel to the normal vector as the normality rule indicates. On the contrary, for the high stress level the direction tilts from the normal vector, especially in the compression - tension stress state. This is the result of the anisotropic behavior of concrete.

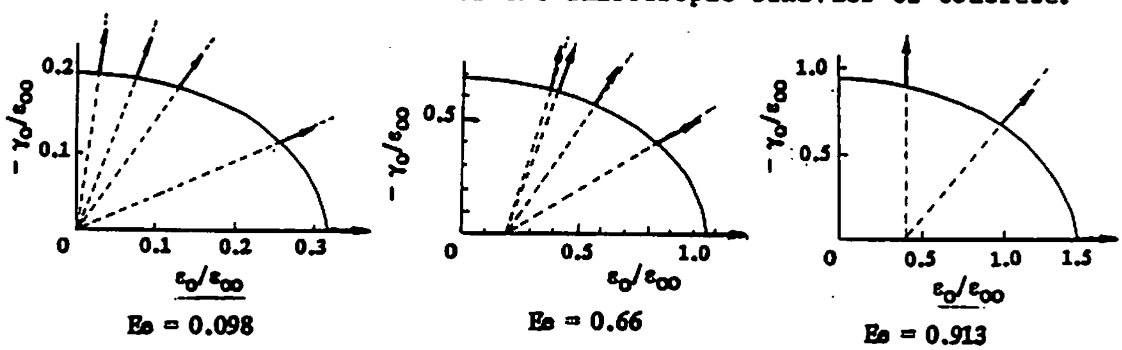

Fig.5 The Direction of Component X2 on Strain Space

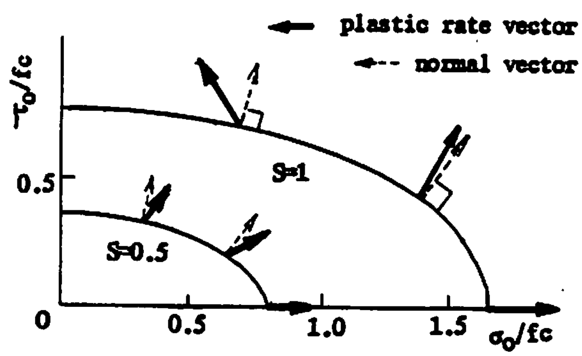

Fig.6 The direction of Plastic Rate Vector

ENDOCHRONIC DESCRIPTION OF SAND BEHAVIOR

Han C. Wu
Professor of Civil Engineering
The University of Iowa
Iowa City, Iowa 52242, U.S.A.

The endochronic theory of plasticity [1] has been applied to describe the mechanical behavior of drained sand. In this application, the endochronic constitutive equations have been modified to account for special features of sand behavior [2]. They read

$$\underset{\sim}{\varepsilon} = \frac{1}{3}\left(A\,\sigma + \sum_{h} B^h q^h + \sum_{d} C^d q^d\right)\underset{\sim}{\delta} + E_2\,\underset{\sim}{s} + \sum_{s} F_2^s\,\underset{\sim}{p}^s \qquad (1)$$

in which $\underset{\sim}{\varepsilon}$ is the strain; σ is the hydrostatic stress; $\underset{\sim}{s}$ is the deviatoric stress; $\underset{\sim}{\delta}$ is Kronecker's delta; q^h, q^d, and $\underset{\sim}{p}^s$ are r number of internal variables such that $r = h + d + s$; A, B^h, C^d, E_2, and F_2^s are material constants.

The values of these internal variables grow as the internal structures of materials change by deformation. The r internal variables are divided into three groups, each being active only for a specific function. Thus, h number of internal variables q^h are active in the representation of hydrostatic response, s internal variables $\underset{\sim}{p}^s$ are responsible for deviatoric response and d number of internal variables q^d are related to volume change (densification or dilation) due to shearing. The last group, q^d, distinguishes granular materials from other continua such as metals or concrete.

The rate of change (or evolution) of each group of internal variables is a material property and depends on the material that the model is presumed to describe. Therefore, the evolution equation may be linear or nonlinear depending on the material at hand. In [2], it has been found that a nonlinear evolution equation for q^h, and linear evolution equations for $\underset{\sim}{p}^s$ and q^d are satisfactory for the sands considered.

The evolution equations may be integrated resulting in expressing the current state of internal variables in terms of deformation histories leading up to the present state. By substituting these expressions for the internal variables into Eq. 1, and thereby eliminating the internal variables from the equation, an explicit equation relating the current stress to the deformation history is obtained. It is remarked that the form of this equation is highly dependent upon the form of the evolution equations, even though the internal variables do not appear explicitly in this equation.