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CONSTITUTIVE EQUATIONS FOR CONCRETE UNDER BIAXIAL STRESS STATES

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ABSTRACT

The multiaxial constitutive equations of concrete play an important part of analysis by the Finite Element Method, especially in the plane stress analysis of 2D reinforced concrete structures. The primary objective of this research is to establish the mathematical description of mechanical deformation behavior of concrete. Biaxial compression-tension tests were executed from which the elasto-plastic and fracture model was derived. With fracture paramter and equivalent plastic strain, this model represents the damage condition of concrete applied by external action (stresses). Theses scalar values are path-dependent, so that this constitutive equations can be used for problems including repeated and non-proportional loadings.

INTRODUCTION

The stress condition of allmost all the parts of reinforced concrete members are considerd to be under biaxial compression-tension stresses. Especially in case of 2D reinforced concrete structures such as beams, plates and corbels, the ultimate strength of structures is often determined by the fracture of concrete under compression-low tension stresses. However, a lot of experimental studies concerned with multiaxial conditions have delt with biaxial or triaxial compressions and in many case with failure function indicated by stresses, so that the accumulation of concrete deformation data with tensile stresses are not sufficient for establishing more precise constitutive law of concrete. As a result, the analytical predictions are not sufficient enough to discuss the advanced mechanics of reinforced concrete structures. It is the objective of this research to make the hehavior of concrete under biaxial tension-compression clear by means of experiments and to establish the plane stress constitutive equations (stress strain relationship) of concrete.

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1. ABSTRACT OF EXPERIMENT

At the same time, two concrete plates (20x20x5cm) were used for a test program of one loading hystory. These plates were set in parallel and tensile and compressive forces were applied to the specimens through steel bearing plates as shown in Fig.1. In order to eliminate the shear friction between the concrete specimens and the loading plates two sheets of teflon (0.5 and 0.1mm thickness) with silicon grease were put on the contact faces. Tensile forces were transmitted from bearing plates to wooden brushes and from the brushes to the specimens. The wooden brushes as shown in Photo.1 were put together with adhesive agent of epoxy resign. It was made sure from preliminaly tests that within the experimental accuracy a concrete plate to which wooden brushes were sticked together could be compressed to nearly -4000 micro strains without any disturbance of stress fields because the stiffness of the brushes is much lower than that of the specimen. Two principal strains were mesured by wire strain guages which were supported and compensated by pai-type strain transducers for large strain range, and the applied stresses were calculated from an electric load cells. The loading speed was held nearly constant, and the loading system as shown in Fig.2 was controlled by a microcomputer. In order to make the stiffness of the loading system including the specimens more rigid and to make it possible to get the strain data under the stable condition, two sets of bearing plates were linked with PC stiffners in each principal stress direction as shown in Fig.1. The concrete specimens were shaped by the most precise steel mould. The faces of the specimens to which tensile force was applied were scraped and planed so as to eliminate the weeker layers introduced by bleeding. In this process, the tensile force was transmitted equally towerd mortar and aggregates. The faces to which compressive force was applied were coated with parafin so as to prevent silicon grease from inpregnation into the concrete specimens due to compressive force.

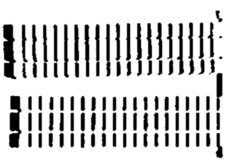

Photo.1 Wooden Brush

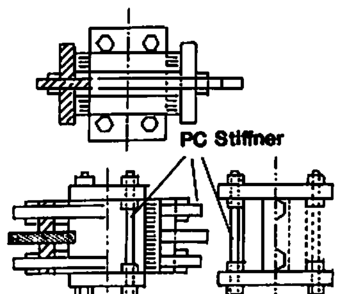

Fig.1 Loading Plates and Specimens

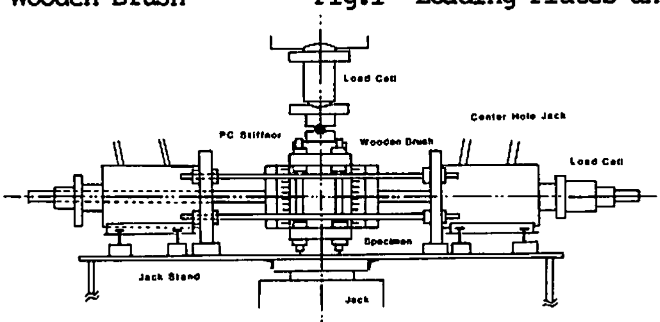

Fig.2 Loading System

2. LOADING PROGRAMS AND RESULTS OF EXPERIMENTS

Loading hystories indicated by stresses were determined by combining the following two types of loading.

A) $d\sigma_1 \neq 0$ under $d\sigma_2 = 0$
B) $d\sigma_1 = 0$ under $d\sigma_2 \neq 0$
where $d\sigma_1$, $d\sigma_2$: maximum and minimum principal stresses

Loading programs included perfect unloading pathes on stress space in order to mesure the plastic strain increments. From the experimental setup two principal stress directions were fixed. In this paper, stress and strain values are normalized by F1 and Epu. The former is the uniaxial compressive strength (peak stress point) and the latter, the strain corresponding to F1.
The monotonic loading results of experiments are shown in Fig.3, where ε_1 and ε_2 indicate the maximum and minimum principal strains. This diagram shows the stress loading pathes and biaxial stress strain relationship. Under high compressive stresses, tensile stress increments orthogonal to the compressions cause large deformations not only in the direction of the maximum principal stresses but also of the minimum stresses. The values of the peak stresses and the corresponding strains are not path-dependent on monotonic loading data.

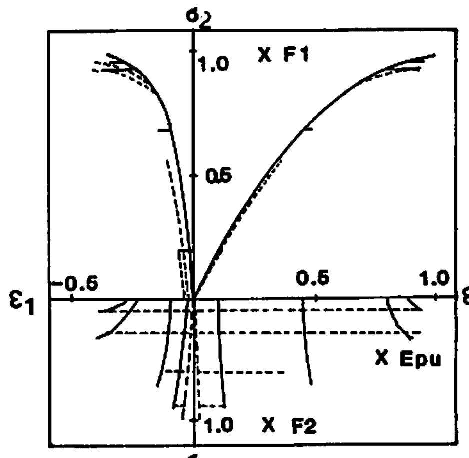

Fig.3 Biaxial Stress Strain Curves

3. BASIC PHILOSOPHY OF ELASTO-PLASTIC AND FRACTURE MODEL

When external actions are applied to concrete, plastic deformation appears, and microcracking and local buckling propergate so that stress strain relationship under monotonic loading represents the nonlinearlity and unloading stiffness decreases with increasing deformation. These characteristics can be modeled by the following assumptions.

(a) Concrete is the composite material composed of the parallel constituent elements which behave as an elasto-plastic strain hardening material as shown in Fig.5.

(b) Each constitutive element loses its resistance capacity against the external stress when the element stress reaches to the specific strength. This process is the irrecoverable one. This assumption represents the local buckling due to microcrack propergation.

(c) The specific strength of each element distributes and there exists the probarbility distribution of strength as shown in Fig.4. This modelling means that concrete is not a uniform material but has the geometrical ununiformity of strength.

Theese basic assumptions give the idealized stress-strain diagrams for composite materials as shown in Fig.5. $\bar{\varepsilon}$, $\bar{\varepsilon}el$, $\bar{\varepsilon}pl$ and $\bar{\sigma}$ are total strain, elastic strain, plastic strain and total stress level, respectively. In the case (I) of Fig.5 where the total strain level is low , the progress of plasticity and fracture is very little so that stress-strain relationship as a composite material is nearly the same as that of linear one. When the total strain level increases and each element stress becomes large (II) by strain hardening plasticity, some constituent elements reach to the fracture strength and permanently lose their stiffness and resistance capacity. As a result, the unloading stiffness of concrete as a composite becomes small. The existence probarbility of concrete elements defined as K is calculated from the strength distribution P shown in Fig.4 and Equation (1).

$$K = 1 - \int_0^f P(l)dl \qquad f = E_0(\bar{\varepsilon}max - \bar{\varepsilon}pl) \qquad (1)$$

When deformation becomes larger, rapidly proceeds the fracture of constituent element which cancels the increase in element stresses due to strain hardening so that the average stress as a composite material, that is, the summation of existent elements' stresses from the elasto-plastic and fracture model gradually decreses. This characteristics is called strain softening. The stress-strain relationship of concrete conposite can be formulated as follows.

$$Si = E_0(\bar{\varepsilon} - \bar{\varepsilon}pl) \qquad (2) \qquad S = \sum Si = E_0 K(\bar{\varepsilon} - \bar{\varepsilon}pl) \qquad (3)$$

where E_0 : elastic stiffness of each element Si : stress of element
K : fracture parameter

Equation (3) is equivalent to the basic assumptions (a)-(c) which are able to describe the behaviors in irrecoverable process (loading) and recoverable one (unloading), resectively. K and Epl are considered to be scalar values which are path-dependent. For expanding this concept to the more general stress states, plastic, elastic and fracture parameter must be determined under biaxial stress and strain tensors.

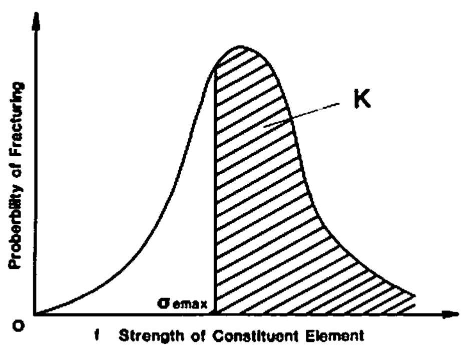

Fig.4 Strength Distribution

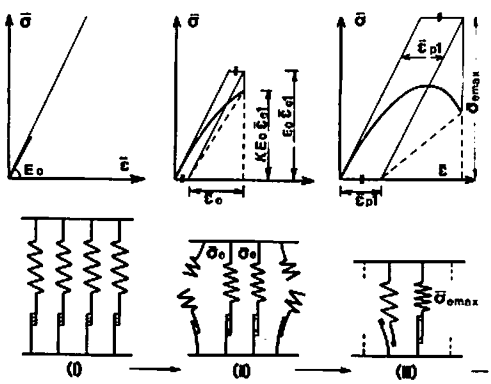

Fig.5 Basic Concepts of Elastic Plastic and Fracture Models

4. EQUIVALENT STRESS

The equivalent stress which is the scalar value and represents the stress level under biaxial stresses must be a function of stress invaliants $\bar{\sigma}$ and $\bar{\tau}$, which are the mean stress and the deviatric stress under plane stress tensorial system respectively.

$$\bar{\sigma} = \sqrt{2\left(\frac{\sigma_1 + \sigma_2}{2}\right)^2} \qquad (4) \qquad \bar{\tau} = \sqrt{2\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2} \qquad (5)$$

Let us now consider the isotropic stress condition ($\bar{\tau}=0$). In this case, the value of stress tensor is constant during coordinate transformation so that it is suitable to define the stress level as expressed by Equation (6).

$$S = a\,\bar{\sigma} \qquad a = \text{const.} \qquad \text{isotropic condition} \qquad (6)$$

When the deviatric stress is added to the isotropic stress states ($\bar{\tau} \neq 0$ $d\bar{\tau}=0$), it is easily understood from experimental results that the plastic strain and fracture progress and that new damage is accumulated in concrete. In this case the equivalent stress should be evaluated higher than the isotropic condition. From the upper point of view, the authors propose the equivalent stress S as Equation (7).

$$S = \sqrt{(a\bar{\sigma})^2 + (b\bar{\tau})^2} \qquad a=0.61 \quad b=1.276 \qquad (7)$$

S means the 'length' on the stress space, and the coefficients a and b represent the weight values which indicate the degree of contribution of each invariant to the stress level. The plotted data in Fig.6 have the meaning of the peak strength points on $\bar{\sigma}-\bar{\tau}$ space. Two weight values were determined so that the curve corresponding to S=1 may envelope the data with compression failure mode in Fig.6. The definition of this equivalent stress assumes that stress state or stress level on the failure envelope is imagined to have the same condition and that the weight values are effective not only for the peak stresses but for any stress values. Actually the stress points on compression mode failure envelope corresponds to the balanced points of increasing stress of each element due to strain hardening and decreasing stress by fracturing as a composite material.

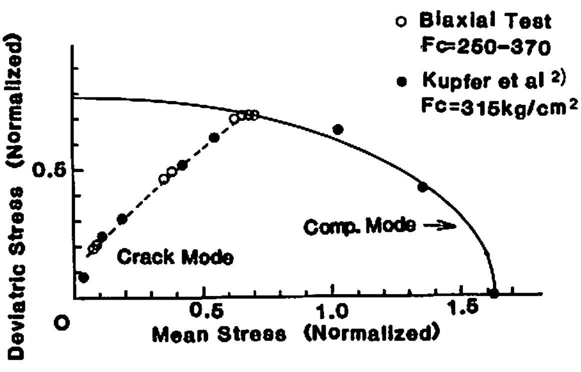

Fig.6 Fialure Envelope (Stress)

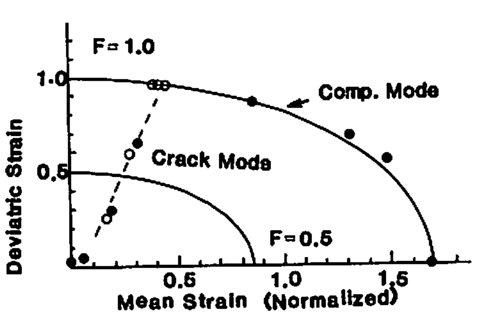

Fig.7 Failure Envelope (Strain)

5. EQUIVALENT STRAIN

Total strain tensors can be devided into two parts, elastic and plastic ones as Equation (8).

$$\varepsilon_{ij} = \varepsilon_{ijel} + \varepsilon_{ijpl} \qquad (8)$$

In order to introduce the scalar values which indicate the elastic and plastic strain levels, the following function is defined.

$$F = F(\delta_{ij}) = \sqrt{(c\bar{\epsilon})^2 + (d\bar{\gamma})^2} \qquad c=0.589 \qquad d=1.01 \qquad (9)$$

$$\bar{\epsilon} = \sqrt{2\left(\frac{\delta_1 + \delta_2}{2}\right)^2} \qquad (10) \qquad \bar{\gamma} = \sqrt{2\left(\frac{\delta_1 - \delta_2}{2}\right)^2} \qquad (11)$$

Function F is indicated by two-dimensional tensor invariants. The weight coefficients were determined by the same procedure as in case of the equivalent stress. Fig.7 shows the strain points corresponding to the stress data in Fig.6. In this model, function F is used for scale unit which means the 'length' on the strain space. The equivalent elastic strain ,in other words, the deformation level of elasticity for concrete constituent elements is defined using 'strain mesure function' as follows.

$$Eel = F(\varepsilon_{ijel}) = F(\varepsilon_{ij} - \varepsilon_{ijpl}) \qquad (12)$$

Furthermore, the equivalent plastic strain which represents the plastic deformation level must be introduced. From the physical point of view, the equivalent plastic strain cannot be formulated by the integrated form like Equation (12), because the plasticity is the irrecoverable and path-dependent behavior, so that the definition of plastic strain level must be described by differencial forms. The authors propose the following defenition for equivalent plastic strain which is the modification of well-known effective strain of work hardening plasticity.

$$Epl = \int dEpl \qquad dEpl = \left.\frac{\partial F}{\partial \delta_{ij}}\right|_{\delta_{ij}=\varepsilon_{ijel}} d\varepsilon_{ijpl} \qquad (13)$$

From the basic concept, the level of total deformation has the following forms.

$$E = \int dE = \int dEel + \int dEpl \qquad (14)$$

for recoverable process $\qquad dE = dEel = \frac{\partial F}{\partial \delta_{ij}}(d\varepsilon_{ij} - d\varepsilon_{ijpl}) = \left.\frac{\partial F}{\partial \delta_{ij}}\right|_{\delta_{ij}=\varepsilon_{ijel}} d\varepsilon_{ij} \qquad (15)$

for irrecoverable process $\qquad dE = dEel + dEpl = \frac{\partial F}{\partial \delta_{ij}}(d\varepsilon_{ij} - d\varepsilon_{ijpl}) + \frac{\partial F}{\partial \delta_{ij}} d\varepsilon_{ijpl}$

$$= \left.\frac{\partial F}{\partial \delta_{ij}}\right|_{\delta_{ij}=\varepsilon_{ijel}} d\varepsilon_{ij} \qquad (16)$$

The equivalent total strain has the same definitions under loading and unloading conditions. This coincidence makes the analytical formulation simple and useful.

6. PROPERTIES OF PLASTIC FLOW UNDER BIAXIAL STRESSES

From these expressions of Equation (12)-(16), E,Epl and Eel can be calculated using experimental data by finite differencial method. There exists one to one relationship between the equivalent plastic strain Epl and maximum total equivalent strain Emax as shown in Fig.8. The mathematical description between Epl and Emax is given as Equation (17).

$$Epl=(0.83E_{max}^{4}-0.82E_{max}^{3}+0.64E_{max}^{2})/(0.83E_{max}^{3}+0.18E_{max}^{2}+0.64E_{max}+1.96) \quad (17)$$

From the basic assumptions, the maximum elastic strain level corresponds to the applied maximum stress level Si,max which spreads out the plastic potential by strain hardening. The value dEel/dEpl has the meaning of hardening coefficient of plasticity.

7. PROPERTIES OF FRACTURE UNDER BIAXIAL STRESSES

The fracture parameter K has the following form which is derived from the elasto-plastic and fracture model.

$$K = S/E_o(E - Epl) \; , \quad E_o=2.37 \quad (18)$$

In other words, fracture parameter K represents the average unloading stiffness. From the experimental data, the one to one relationship is observed between K and Emax as shown in Fig.9, and mathematical relation is given by Equation (19).

$$K = EXP(-0.66E_{max}^{1.02}) \quad (19)$$

Equivalent plastic strain and fracture parameter are the values which describe the inner conditions of concrete. Theses values can not be calculated by the update strains, but can be evaluated by time integration that is, by knowing all the informations about strain hysteresis.

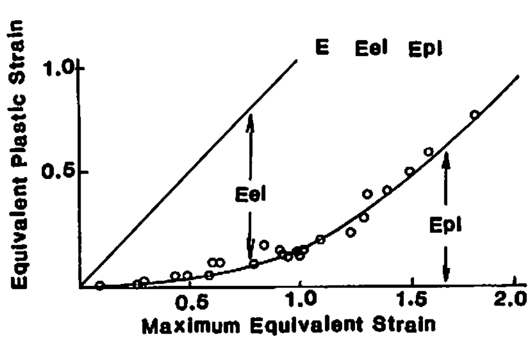

Fig.8 Equivalent Plastic Strain

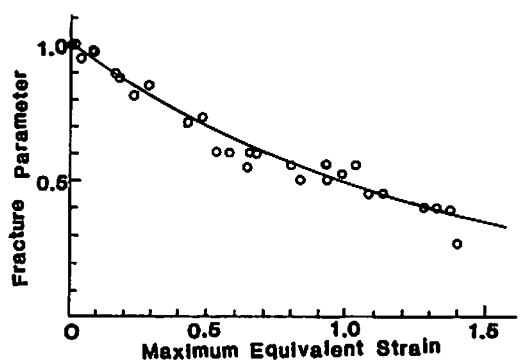

Fig.9 Fracture Parameter

8. EQUIVALENT STRESS STRAIN RELATIONSHIP

Under the condition where the principal axis is fixed, equivalent stress and strain can be connected as Equation (20).

$$S=2.37\exp(-0.66E_{max}^{1.02})\{E-(0.83E_{max}^4-0.82E_{max}^3+0.64E_{max}^2)/(0.83E_{max}^3+0.18E_{max}^2+0.64E_{max}+1.96)\} \quad (20)$$

Equation (20) gives the stress strain relationship in monotonic, unloading and reloading processes. In the monotonic loading, equivalent total strain E is always equal to Emax. Fig.10 shows the experimental data and analytical prediction. With reasonable accuracy analytical model can predict the behavior of concrete under any loading hysteresis.

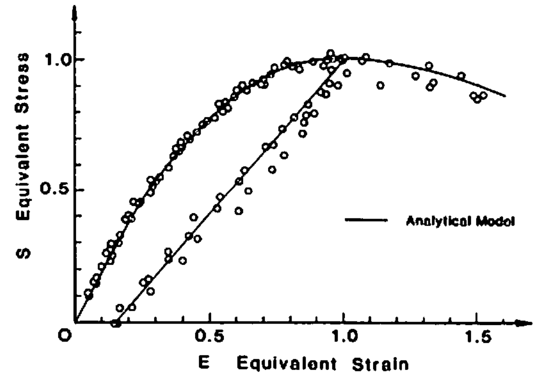

Fig.10 Equivalent Stress Strain relationship

CONCLUSION

The mechanical deformation of concrete can be modeled by elasto-plastic and fracture model, and under the biaxial stress states the plastic and fracture conditions are uniquely described by scalar values Epl and K which are path-dependent but not on coordinate transformation. However, this model does not evaluate factors of the strain rate and unloading nonlinearity because of simplicity.

In order to descide the 2D stresses when the strain hystory is given, a flow rule which defines the relationship between the direction of stresses and that of plastic or total strains must be added to the elasto-plastic and fracture model.[3][1]

AKNOWLEDGEMENT

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