This paper is a dedication to the 60th birthday of Prof. Dr. Bruno Thürlimann

FATIGUE FAILURE IN SHEAR OF BEAM WITHOUT WEB REINFORCEMENT
--- INFLUENCE OF LOAD RANGE ON FATIGUE STRENGTH IN SHEAR ---

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1. INTRODUCTION

A reinforced concrete beam sometimes fails in shear under fatigue loading due to the fracture of web reinforcement even if the applied maximum shear force is much less than the ultimate static strength [1][2]. Since the fatigue fracture of web reinforcement depends on the stress intensity, it is imperative first of all to investigate the characteristics of stress under fatigue loading. Recently, the authors have reported that stirrup strains increased the cycles of repeated loading and proposed an equation for the calculation of stirrup strain by applying the fatigue strength in shear of the identical beam without shear reinforcement to the decrease of shear force carried by concrete [1].

The experimental researches on fatigue strength of beams without shear reinforcement have made clear that fatigue strength at 1 mega cycle is about 60% of the static strength in the cases of a large span depth ratio [2][3][4][7], the so-called S-N curves of beams with a large span depth ratio are different from those with a small span depth ratio [4], and that the beam, which should fail in flexure under static loading, sometimes fails in shear if subjected to fatigue loading [7]. However, the influences of load range and size of specimen on the fatigue strength have not yet been reported. Consequently, no S-N curve which includes these factors has been reported. Therefore, fatigue tests of sixteen rectangular beams were carried out. The main parameters were the load range and the height of beams.

2. FATIGUE STRENGTH WITHOUT THE INFLUENCE OF LOAD RANGE

The evaluation of static strength is important, when the fatigue strength is represented by the ratio of the applied maximum shear force to the static strength. Equation 1 is used for the calculation of the static strength. This equation can estimate the static strength with less than 10 % of the coefficient of variation [8], which is considered accurate enough for this purpose.

$$V_{cu} = 0.20\, f_c'^{1/3}\,(0.75 + 1.40\, d/a)\,(1 + \beta_p + \beta_d)\, b_w\, d \quad \text{-----} (1)$$

where f_c' : concrete compressive strength
β_p : $(100\, p_w)^{1/2} - 1$
p_w : $A_s/b_w d$ = reinforcement ratio
β_d : $(1000/d)^{1/4} - 1$
b_w : web width [mm]
d : effective depth [mm]
a : shear span

Table 1. Test specimens and test results

Specimens	a (mm)	d (mm)	b (mm)	p_w (%)	f_c' (MPa)	V_f 1)(kN)	V_{cu} (kN)	V_{max} (kN)	V_{min} (kN)	V_{min}/V_{max}	V_{max}/V_{cu}	N_f (×10³)
1a	1540	440	200	0.68	33.4	65	69	49	4.9	0.1	0.72	0.05
1b	1540	440	200	0.68	33.4	65	69	49	30	0.6	0.72	314 0
2a	1540	440	200	0.68	45.5	66	76	46	4.6	0.1	0.61	1.86
2b	1540	440	200	0.68	45.5	66	76	46	28	0.6	0.61	(447.9)
								74			0.97	1)
3a	1540	440	200	1.67	33.4	138	99	60	24	0.4	0.61	(1 032.0)
								62			0.83	2)
3b	1540	440	200	1.67	33.4	138	99	71	36	0.5	0.72	0.07
4a	1540	440	200	1.67	45.5	143	110	67	6.7	0.1	0.61	43.0
4b	1540	440	200	1.67	45.5	143	110	79	32	0.4	0.72	0.23
5a	770	220	400	0.68	34.2	65	85	58	5.8	0.1	0.69	[24.5]
5b	770	220	400	0.68	34.2	65	85	58	23	0.4	0.69	34.1
6a	770	220	400	0.68	46.0	66	93	57	5.8	0.1	0.61	[31.1]
6b	770	220	400	0.68	46.0	66	93	57	23	0.4	0.61	(312.1)
								62	25	0.3	0.61	(23.15)
								67	34	0.5	0.72	36.9
7a	770	220	400	1.67	34.2	139	115	98	59	0.6	0.85	0.049
7b	770	220	400	1.67	34.2	139	115	98	9.8	0.1	0.85	0.024
8a	770	220	400	1.67	46.0	143	127	99	79	0.8	0.78	70.6
8b	770	220	400	1.57	46.0	143	127	108	97	0.9	0.85	(398.4)
								108	86	0.8	0.85	(387.0)
								117	94	0.8	0.92	(123.5)
								117	70	0.6	0.92	0.267

Notations for Tables 1 and 2.

Vmin : applied minimum shear force

( ) : non-failure

1) : Flexural failure under static test after fatigue test

2) : Shear failure under static test after fatigue test

[ ] : Fatigue flexural failure due to fatigue fracture of tensile bar

Table 2. Fatigue lives of specimnes (one thousand cycles)

Vmax/Vcu \ Vmin/Vmax	0.1	0.4	0.5	0.6	0.8	0.9
92 %				2.7	1235	
85 %	0.2			0.5	3870	1984
78 %					706	
72 %	0.5	2.3	0.7, 369	3140		
69 %	[245]	341				
66 %		232				
62 %	28.6, 432, [3[illegible]]	(1020), (3121)		3448		

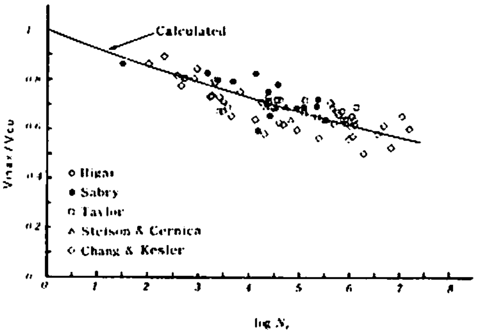


Fig. 1. Fatigue strength in shear of beam without web reinforcement normalized with static strength, Vcu, calculated by Eq. 1 [2] [3] [4] [5] [6] [7]

Figure 1 shows that the relationship between the ratios of the previous tested fatigue shear strengths to the values calculated by Eq.1 and the tested fatigue lives of the beams. The calculated static strengths are multiplied by 0.8 for the cases of light weight concrete. The solid line in Fig.1 is a calculated line by the following equation, which does not include the effect of load range.

$$\log(V_{max}/V_{cu}) = -0.035 \log N_f \quad \text{------(2)}$$

where Vmax : applied maximum shear force

Vu : static strength in shear of beam without web reinforcement

Nf : fatigue life

The average ratio of the tested maximum shear forces to the calculated shear forces is 1.00 and the coefficient of variation is 7.4 %.

3. OUTLINES OF TESTS TO CLARIFY THE INFLUENCE OF LOAD RANGE

Fatigue tests of sixteen rectangular beams without web reinforcement shown in Fig.2 were carried out. The load range and the heigth of beams were the main parameters (see Table 1). For eight specimens the effective depth was 440 mm which was much larger than that of 108 to 220 mm in the previous tests [2]-[7]. Concentrated load was applied at two points to make a/d equal to 3.5. The static flexural strength was larger than the static shear strength in the specimens with reintorcement ratio of 1.67%, while the static shear strength was larger in the specimens with reinforcement ratio of 0.68 %.

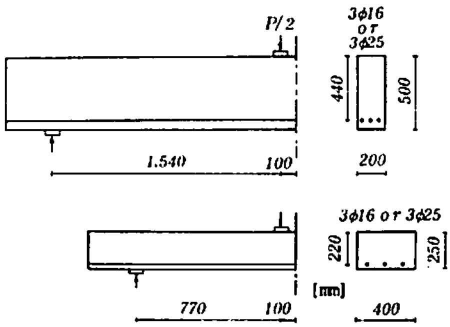


Fig. 2. Rectangular beams without web reinforcement

Two batches of ready mixed concrete with maximum aggregate size of 25 mm were used. The cylinder strengths were obtained as shown in Table 1. All the specimens were placed in the laboratory more than one month before the tests. The fatigue tests were carried out between sixty and one hundred fifty days after casting. All the bars were deformed bars and had two longitudinal ribs and parallel transverse lugs perpendicular to the bar axis. The diameters were 25 mm for reinforcement ratio of 1.67% and 16 mm for reinforcement ratio of 0.68%, and the cross-sectional areas were 1,470 and 600 mm^2 respectively. The yield strength of the bar with the diameter of 25 mm was 370 MPa and that of the bar with the diameter of 16 mm was 400 MPa.

Each specimen was subjected to the constant maximum and minimum load. The applied maximum shear force was 61 to 92% of the static shear strength. The applied minimum shear force was changed widely between 10 and 90% of the maximum one to investigate the influence of load range. The apparatus for loading consisted of a steel frame and a hydraulic jack connected to a pulsator. All the specimens were loaded statically during the first hundred cycles and after that they were loaded dynamically at 210 cycles per minute with sine loading curve until the specimens failed. Four specimens which did not fail under repeated loading were made to fail due to static loading or by changing the level of the repeated loading.

4. FATIGUE STRENGTH INCLUDING THE INFLUENCE OF LOAD RANGE

Load levels and fatigue lives of specimens tested by the authors are shown in Table 1. Twelve of the sixteen specimens failed in shear due to the propagation of the main diagonal crack. Two of the remaining specimens failed in flexure due to the fatigue fracture of the tensile bars at the maximum moment region and the other two failed under the static loading after the fatigue tests.

There are three pairs of the specimens, 1a-1b, 2a-2b and 7a-7b, which are identical except for the magnitude of the applied minimum load. The fatigue life of one specimen, with a larger ratio of the minimum load to the maximum, was larger than that of the other. And the specimen 8b did not fail under the third repeated loading, but failed under the fourth repeated loading whose minimum load was smaller than that of the third one without change of the maximum load. The test results excluding those of the beams failing in flexure are rearranged in Table 2 to confirm the influence of load range. The tested values of fatigue lives are classified according to the value of the ratio of the applied maximum shear force to the static shear strength, Vmax/Vcu. This table shows that the smaller the ratio of the applied minimum shear force to the maximum, Vmin/Vmax (called hereafter 'r'), the shorter the fatigue life between the specimens with the same Vmax/Vcu ratio. Consequently, it can be said that the larger the load range, the shorter the fatigue life.

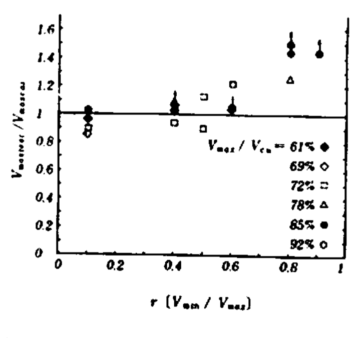
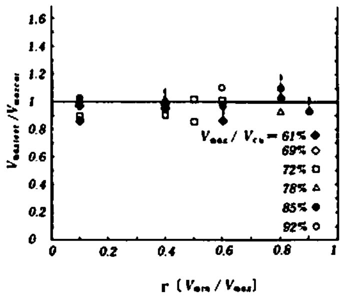
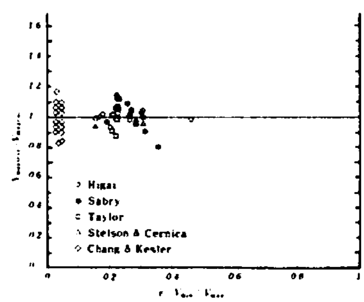


Fig. 3. Influence of ratio, r, of minimum shear force to maximum shear force on fatigue strength in shear of beam without web reinforcement

The relationship between the ratio, r, and the ratio of the tested value of fatigue strength to the value, Vmaxtest/Vmaxcal, calculated by Eq.2 which was proposed without any consideration of the influence of load range is shown in Fig.3. The calculated values have a tendency to become smaller than the tested ones with increase of the r values.



(a) Authors' tests



(b) Reported data [2] [3] [4] [5] [6] [7]

Fig. 4. Relationships between the ratio of tested value or fatigue strength in shear of beam without web reinforcement to that calculated by Eq. 3 and ratio, r, of minimum to maximum shear force

Although the fatigue strength can be evaluated from Eq.2 in the case of r smaller than about 0.5, it cannot be evaluated in the case of r larger than 0.6. From this result it can be supposed that the influence of load range appeared not so clearly from the previous tests where most of the values of r were smaller than 0.5. Finally, with consideration of the influence of load range the following equation for the prediction of the fatigue shear strength is proposed.

$$\log(V_{max}/V_{cu}) = -0.036(1-r^2)\log N_f \text{ ------(3)}$$

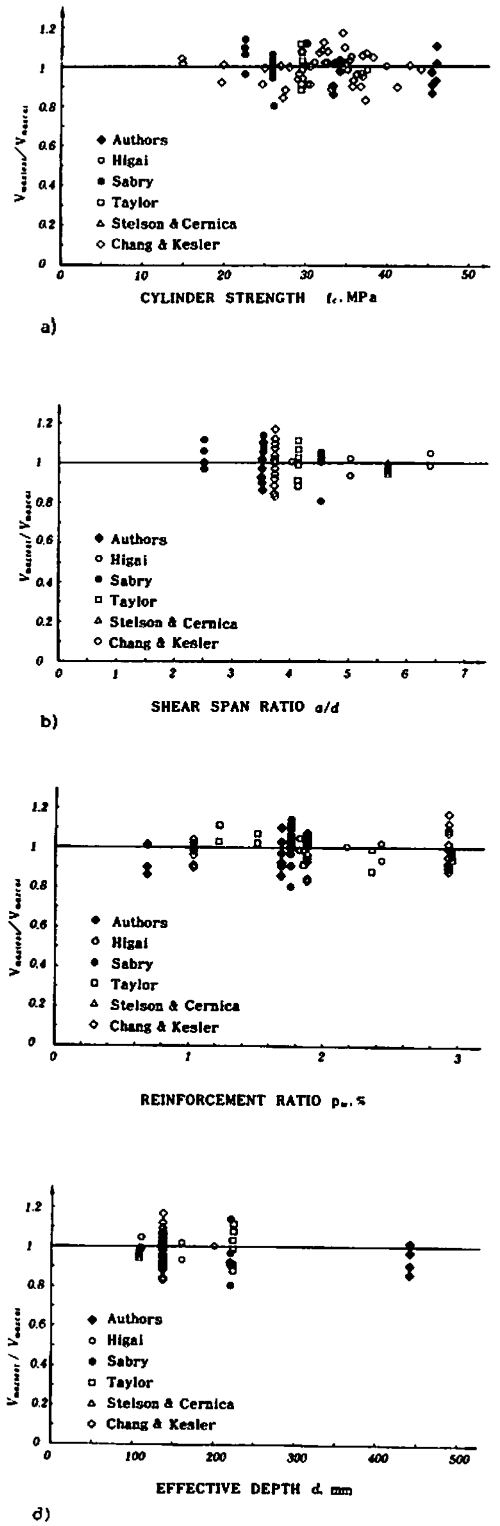
where r = Vmax/Vmin

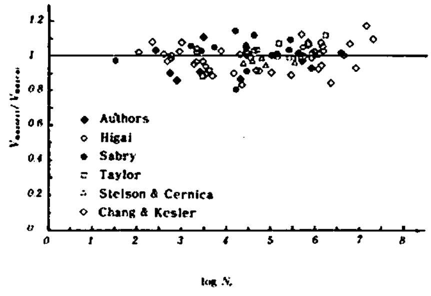
The relationship between r and the ratio of the tested value of the fatigue strength to that calculated by Eq.3 is shown in Fig.4(a). Figure 4(b) shows the ratio of the previously tested value of the fatigue strength [2]- [7] to the calculated fatigue strength. The average is 0.99 and the coefficient of variation is 7.4 %.

The relationship between the value of Vmaxtest/Vmaxcal and the cylinder strength, the ratio of shear span to effective depth, the reinforcement ratio and the effective depth which are parameters for the calculation of the static shear strength are given in Fig.5(a)(b)(c)(d). The relationship between the value of Vmaxtest/Vmaxcal and the tested value of fatigue life is given in Fig.5(e). The values of Vmaxtest/Vmaxcal are not correlated to any parameter. This means that Eq.3 can be used for the evaluation of fatigue shear strength of beams without shear reinforcement.

5. DECREASE OF SHEAR FORCE CARRIED BY CONCRETE IN THE BEAM WITH WEB REINFORCEMENT UNDER FATIGUE LOADING

The increase of stirrup strain under fatigue loading has been explained by the assumption of the decrease of shear force carried by concrete [1]. The rate of the decrease of shear force carried by concrete can be considered to be almost constant in the cases of different magnitudes of the applied maximum and minimum shear force levels. However, the rate of the decrease was reported to be exceptionally smaller in the case where the ratio of the applied minimum shear force to the maximum was close to 1.0. The decrease of shear force carried by concrete can also be considered to be influenced by the ratio r.





(e) Fatigue life, Nf

Fig. 5. Relationships between the ratio of the tested value of fatigue strength in shear of beam without web reinforcement to that calculated by Eq. 3 and various factors

Assuming that the decrease of shear force carried by concrete was essentially the same as that of fatigue strength in shear of beam without web reinforcement, the following equation was derived from Eq. 3 to calculate the average strain of stirrups at the applied maximum shear force under the fatigue loading.

$$\varepsilon_{wmax} = \frac{\beta_x \left[V_{max} - V_{co}\, 10^{-0.036(1-r^2)\log N}\right]}{A_w E_w z/s} \quad -(4)$$

where ε_{wmax}: average strain of stirrups at applied maximum shear force

β_x : average coefficients to cover the influence of support and loading point on reducing the stirrup strain [1]

A_w : cross-sectional area of a pair of stirrups

E_w : Young's modulus of stirrup

z : arm length of the assumed truss

s : spacing of stirrups

V_{co} : shear force carried by cocrete at initial loading

The test results reported previously [1][9] were confirmed to be expressed by Eq. 3 even in the cases where the ratio r was close to 1.0, although the equation proposed previously was not available in these cases. Both of the tested values and those calculated by Eq. 3 in the cases of r equal to 0.09 and 0.82 are shown in Fig.6. The calculated values were seen to express appropriately the variation of the rate of strain increase due to the variation of r.

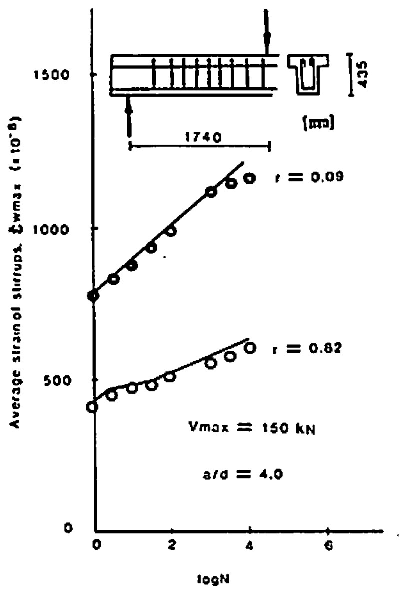


Fig. 6. Comparison between the tested value of average strains of stirrups under repeated loading and that calculated by Eq. 4 in case of different r, ratios.

CONCLUSIONS

(1) The test results show that load range influences the fatigue strength in shear of beam without web reinforcement. The larger the ratio r, of the applied minimum to the maximum shear force, the longer the fatigue life of beam. When the ratio r is larger than 0.6, the longevity of the fatigue life becomes more noticeable.

(2) The fatigue strength in shear of beam without web reinforcement in the case of a relatively large a/d can be predicted by using Eq. 3 proposed with consideration of the influence of load range. This equation can also be applied for the previously reported data.

(3) The decrease of shear force carried by concrete, which can explain the phenomenon of strain increase in stirrups under fatigue loading can be assumed to be the same as that of the fatigue strength in shear of beam without web reinforcement, which is expressed by Eq. 3 . The equation derived from this assumption can nicely evaluate the increase of stirrup strains, even in the case of the ratio r close to 1.0.

REFERENCES

[1] Okamura, H., Farghaly, Sabry A. and Ueda, T. : Behaviors of Reinforced Concrete Beams with Stirrups Failing in Shear under Fatigue Loading, Proceedings of the Japan Society of Civil Engineers (JSCE), No.308, April 1981

[2] Chang, T. S. and Kesler, C. E. : Static and Fatigue Strength in Shear of Beams, with Tensile Reinforcement, Journal of ACI, Vol.54, pp.1033-1057, June 1958

[3] Higai, T. : Fundamental Study on Shear Failure of Reinforced Concrete Beams, Proceedings of JSCE, No.279, pp.113-126, November 1978

[4] Farghaly, Sabry A. : Shear Design of Reinforced Concrete Beams for Static and Repeated Loads, Dissertation for the degree of Dr. of Engineering to the University of Tokyo, March 1979

[5] Taylor, R. : Discussion of a Paper by Chang, T. S. and Kesler, C. E. : Fatigue Behavior of Reinforced Concrete Beams, Journal of ACI, Vol.55, pp.1011-1015, March 1959

[6] Stelson, T. E. and Cernica, J. N. : Fatigue Properties of Concrete Beams, Journal of ACI, Vol.55, pp.255-259, August 1958

[7] Chang, T. S. and Kesler, C. E. : Fatigue Behavior of Reinforced Concrete Beams, Journal of ACI, Vol.55, pp.245-254, August 1958

[8] Okamura, H. and Higai, T. : Proposed Design Equation for Shear Strength of Reinforced Concrete Beams without Web Reinforcement, Proceedings of JSCE, No.300, pp.131-141, August 1980

[9] Ueda, T. and Okamura, H. : Behavior of Stirrups under Fatigue Loading, Concrete Journal of Japan Concrete Institute, No.197, May 1981