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Reprinted from

Transactions of the Japan Concrete Institute Vol. 3 1981

[IV-12]

BEHAVIOR OF STIRRUP UNDER FATIGUE LOADING **

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ABSTRACT

Fatigue tests of T-beams with multi-level load ranges were carried out with detail observation of stirrup strains. Nine of eleven specimens failed in shear due to fatigue fracture of stirrups. Based on the observation, the previously proposed equation, which was derived for the calculation of the average stirrup strains from the rectangular beam tests with constant cyclic loads, is generalized. The equation is extended to apply to the case of general variable loading by using equivalent loading cycles, which is founded on a newly developed idea. With the assumption that the applied shear - stirrup strain curve is going to a fixed point at unloading, the accuracy of the equation for the calculation of the stress range is increased. From the calculated average stress range, fatigue strength of beam can be estimated.

INTRODUCTION

Recent design codes for shear which were revised referring to the static test results tend to require less web reinforcement than in the previous codes. This tendency demands the further study on fatigue, because a reinforced concrete beam sometimes fails in shear under fatigue loading due to the fracture of web reinforcement even if the applied maximum shear force is much smaller than the ultimate static strength.

Since the fatigue fracture of web reinforcement depends on the stress range, the characteristics of stress under fatigue loading is firstly to be investigated. Fatigue fracture of web reinforcement is considered to be affected by the local stresses caused by the diagonal crossing of shear cracks. Bending operation of a steel bar makes the fatigue strength smaller. Therefore, fatigue strength of web reinforcement should be investigated in consideration of these effects.

Some previous reports pointed out that (1) stirrup strains increased during fatigue loading [4][5][6], (2) fatigue fracture of stirrup occured [5][7][8] and (3) fatigue strength of stirrup was smaller than that of bar itself [5][8]. However no report which deals with the phenomena systematically and rationally is available. The authors carried out previously the fatigue test of 17 rectangular beams with a constant maximum

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** This is the English translation of the paper published in the Institute's monthly Concrete Journal May 1981.

and minimum load applied to each beam, and observed stirrup strain and the fracture in detail [1]. As a result, it was made clear that the average of stirrup strains at the applied maximum shear force was increasing in proportion to logarithm of the loading cycles and was not influenced by the magnitude of the applied maximum shear force. The phenomena were explained reasonablly by assuming that shear force carried by concrete was decreasing in proportion to logarithm of loading cycles, and the decrease was supplemented with the one carried by the assumed truss. Thus the following equation was introduced [1].

$$\bar{\varepsilon}wmax = \frac{\bar{\beta}x \{ Vmax - Vco (1 - k \log N) \}}{Aw\ Es\ z/s} \tag{1}$$

where $\bar{\varepsilon}wmax$: average of stirrup strains at applied maximum shear force
βx : coefficient for each stirrup in order to cover the influence of support and loading point on reducing stirrup strains
$\bar{\beta}x$: average of βx's
Vmax : applied maximum shear force
Vco : shear force carried by concrete at the first loading
k : constant showing decrease of shear force carried by concrete during fatigue loading (= 0.07)
N : number of loading cycles
Aw : cross sectional area of stirrup within a distance s
Es : Young's modulus of stirrups
z : arm length of truss (= d/1.15)
d : effective depth

It was also concluded that the positions of fatigue fracture of stirrups were at bend and the fatigue strength was about one half of that of its straight portion. A design method for stirrups under fatigue loading was proposed as well as the tentative equation for the calculation of stress ranges in stirrups.

However, the following problems still remained. In order to solve them, this study was carried out.
(1) Applicability of Eq.(1) —— The experiments were limited, so that 1) the applied minimum shear force was made constant, 2) the number of specimens in which Vmax were less than Vco was small, 3) the shape of the cross section was rectangular only, 4) the ratio of shear span to effective depth was 2.5 only, 5) the spacings of stirrups were relatively large compared with the beam depth.
(2) Evaluation of stirrup strain at the applied minimum shear force
(3) Evaluation of stirrup strain under fatigue loading with varied load range
(4) Evaluation of fatigue strength of beam failing in shear

1. OUTLINES OF EXPERIMENT

All the specimens had the same T cross section as shown in Fig.1. Loading points were determined to make shear span-depth ratio 2.0 for the right span and 4.0 for the left. There were seven pairs of stirrups in the right span and nine pairs in the left, so that the spacing of stirrups were relatively small. The specimens were designed so that simultaneous yielding of main bars and stirrups should occur. The shear force at

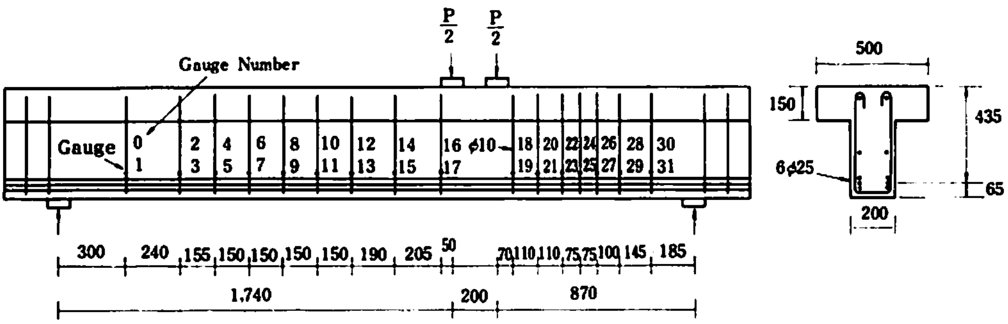


Fig.1 Test specimen

yielding of stirrups was evaluated according to Eq.(2) (see Table 1).

$$Vy = Vco + Aw\ fwy\ (z/s) / \bar{\beta}x \tag{2}$$

where Vy : shear force at yielding of stirrups
fwy : yield point of stirrup

Position of stirrups were determined by using Eq.(2) to produce the same stress in each stirrup under static loading except for the stirrup nearest to the loading point. The latter stirrup was located to make its strain much smaller than the strains in other stirrups.

All the specimens were loaded statically during the first 100 cycles, and after that, loaded dynamically 210 cycles per minute with sine loading curves. A hydraulic jack was used for cyclic and static loading. Details of loading history of each specimen are shown in Fig.2. Two specimens were subjected to a larger load than the maximum of the repeated loading before they were subjected to fatigue loading. All the specimens except for one were subjected to fatigue loading of multi-level. Load ranges were changed with constant minimum load for two specimens and with constant maximum load for two specimens. The minimum load was changed for one specimen with constant load range. For seven specimens the maximum shear force was less than the shear capacity of concrete.

Table 1 Properties of specimens

Specimens	Vco		Vy		Vf(3)	
	Opt. (1) (kN)	Cal. (2) (kN)	a/d=4 (kN)	a/d=2 (kN)	a/d=4 (kN)	a/d=2 (kN)
FS1, FL2 FS3, FL4	96	99	229	428	229	459
FS5, FL6 FS7, FL8	97	101	230	429	231	462
FS9, FL10 FS11	106	106	239	438	235	470

(1) Tested values derived from applied shear - stirrup strain curves
(2) $Vco=0.20fc'^{1/3}(1+\beta p+\beta d)bwd$ (fc':MPa)
where $\beta p=\sqrt{100As/(bwd)}-1$,
$\beta d=(1000/d)^{1/4}-1$ (d:mm)
(3) $Vf=Asfyd(1-0.6pfy/fc')/a$
where bw : web width, a : shear span

All the bars used in the tests were deformed bars having two longitudinal ribs and parallel transverse lugs perpendicular to bar axis. Their material constants are shown in Table 2. The stirrups were bent around the longitudinal bars and

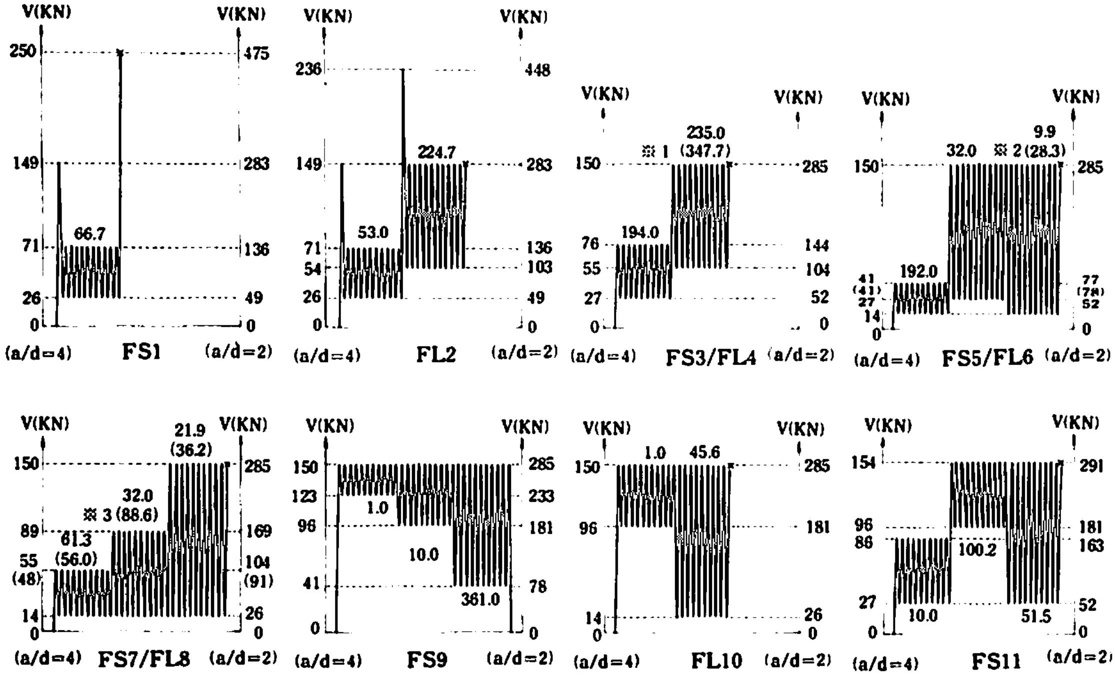


Fig.2 Loading history (*1 FL4, *2 FL6, *3 FL8, Numerals in each figure indicate number of loading cycles x10^4)

Table 2 Properties of materials

Concrete		Stirrup				Longitudinal bar			
fc' (MPa)	M.S. (mm)	(mm)	Aw (mm^2)	fwy (MPa)	Es (10^3MPa)	Φ (mm)	As (mm^2)	fy (MPa)	p (%)
24.3 25.5 29.8	20	10	142.7	383.2	185.7	25	3040	342.2	1.40

fc' : cylinder strength, M.S. : maximum size of coarse aggregate,
Φ : diameter, Aw, As : cross-sectional area,
fwy, fy : yield strength, Es : Young's modulus,
p : steel percentage(=As/(bd)), b : flange width

all the specimens were divided into two groups, called as FS series and FL series respectively, according to the radius of bend. The radius was 1.25 times as much as the diameter of the stirrup in FS series and 2.5 times in FL series. Three batches of ready mixed concrete were used. Four specimens were made from each batch, and compressive strengths at the ages of testing which were between two and twelve months after casting were as presented in Table 2. Electrical resistance strain gauges of 5 mm in length were used for measuring stirrup strains. These gauges were attached to all the stirrups at the position of 50 mm above the center of the longitudinal bars (see Fig.1).

The pulsator was stopped at appropriate loading cycles, and stirrup strains were measured under static loading and the propagation of diagonal cracks was recorded. Concrete cover was removed to confirm the fatigue fracture of stirrup after the test. Widths of diagonal cracks were measured for one specimen by using the contact type gauges. The contact points were put on the surface of concrete in order to make triangles.

2. STIRRUP STRAIN AT THE APPLIED MAXIMUM LOAD

Equation (1), which was previously proposed from the rectangular beam tests with constant minimum load, is confirmed to be applicable to all the T-beam tests with changing minimum load (Fig.3). Fig.4 shows the case in which the minimum load is changed for one beam with constant load range.

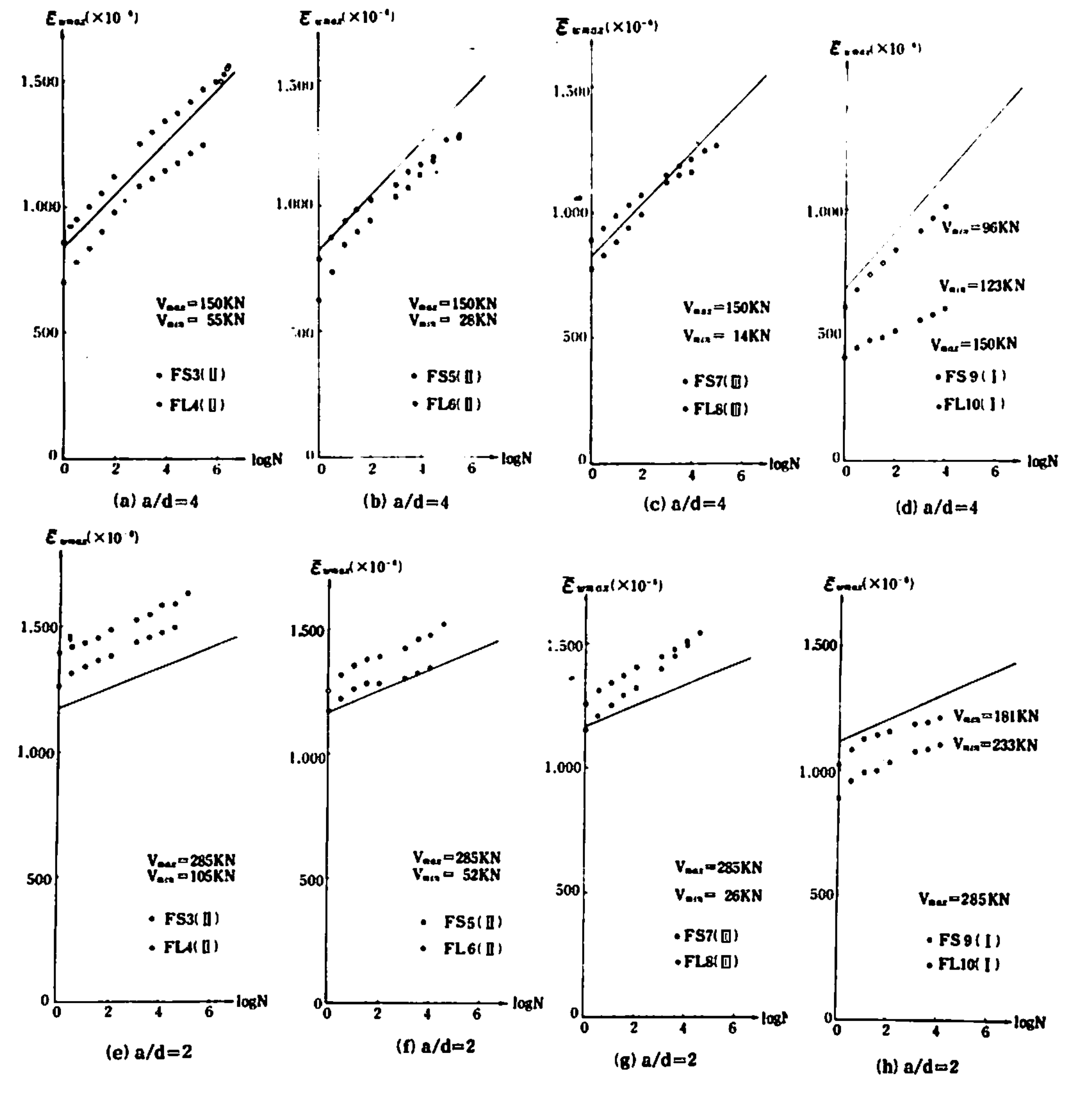


Fig.3 Average of stirrup strains at applied maximum shear force

The solid lines in these figures derived from Eq.(1) agree nicely with tested values. The range of Vmax/Vmin (1.23 to 11.1) and Vmax/Vco (1.42 to 2.94) covers all the practical cases. Eq.(1) is confirmed to be applicable also to the cases in which the influence of support or loading point is either small (a/d = 4) or large (a/d = 2).

The increase of average of stirrup strains in FS9(I), whose value of Vmin is very close to that of Vmax, is smaller than those in other cases as shown in Fig.3(d). When Vmax is equal to Vmin, that is, a beam is subjected to sustained loading, increase of stirrup strain is observed, but much smaller than those subjected to repeated loading (see Sec.4). In conclusion Eq.(1) is generally applicable to reinforced concrete beams except for the cases of extremely small load ranges or the cases influenced by fatigue loading very little.

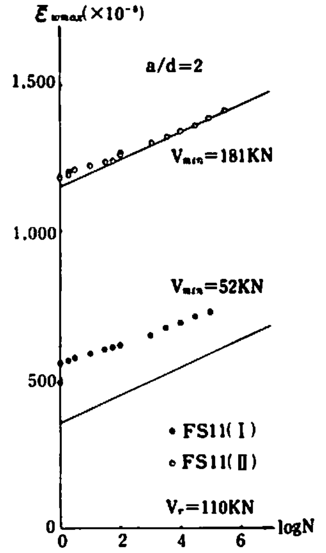


Fig.4 Average of stirrup strains at applied maximum shear force in the case of constant load range

When the applied maximum shear force was less than the shear capacity of concrete (Vmax < Vco), it was observed from the previous tests that stirrup strains hardly increased in the early stage of loading, and stirrup strains began to increase noticeably after some cycles. In the present tests it was observed that the average of strains in stirrups increased little until two or three diagonal cracks occurred in the shear span and one of them was so long to cross some stirrups. The gradual propagation of diagonal cracks during fatigue loading was something like a slow-motion of the propagation under static loading.

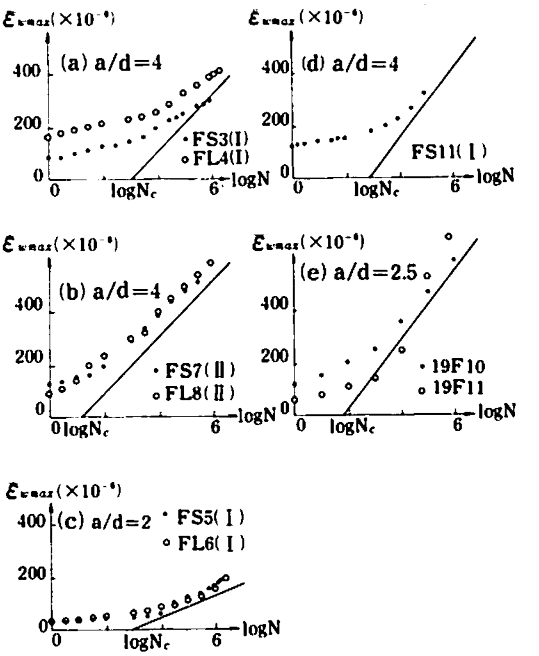


Fig.5 Average of stirrup strains at applied maximum shear force which is less than the shear capacity of concrete

The idea on which Eq.(1) is based can be extended to estimate not only the number of cycles, Nc, when stirrup strains begin to increase but also the increase of the strains thereafter. The value of the shear capacity of concrete is the same as that of shear force, Vco, carried by concrete at the first loading, and decreases in proportion to logarithm of loading cycles until Nc cycles. After Nc cycles the applied maximum shear force Vmax is carried by two components Vs and Vc (the former is shear force carried by the truss mechanism with 45° diagonal struts and the latter is that carried by concrete), and Vc decreases in proportion to logarithm of loading cycles. Eq.(3) is then obtained.

$$Vmax = Vco (1 - ko \log Nc) \quad (3)$$

Eq.(3) is transposed to Eq.(4).

$$\log Nc = (1 - Vmax/Vco) / ko \quad (4)$$

where ko is a constant and assumed to be 0.07. This is equal to the value of k in Eq.(1) which is applicable to the case where diagonal cracks already exsist. The stirrup strain at the applied maximum shear force can be calculated by Eq.(1), substituting for N the total loading cycles from the start of fatigue loading. The calculated values, which are compared with the experimental values in Fig.5, can be said to represent the actual phenomena nicely. These calculated lines agree with the results of both the previous and present tests better than those proposed previously [1].

3. STIRRUP STRAIN AT THE APPLIED MINIMUM LOAD (STRAIN RANGE OF STIRRUP)

In order to make the fatigue strength of stirrup clear, the stress range in stirrup under fatigue loading need be determined. The equation for the calculation of the stress range was temporarily proposed as a result of observation which showed that stirrup stress changed linearly with the change of load [1]. But the equation was incomplete due to the neglected residual strain. In the present study the stirrup strain at the applied minimum load was examined in detail, and a new improved equation was derived.

Fig.6 shows the relationship between average strains, $\bar{\varepsilon}w$, of stirrups in FS5 and applied shear forces, V, at 10 and 10^4 cycles. The area of the loop at 10^4 cycles is smaller than the area at 10 cycles. The shear force at the folded point becomes larger with increase of loading cycles. Although the loop has the folded point, the $\bar{\varepsilon}w$-V curve can be roughly said to be a straight line. The inclination of the straight line becomes smaller with increase in number of cycles of fatigue loading, so that the strain range corresponding to the same range of shear force becomes larger. And a significant increase of the residual strain is observed during fatigue loading.

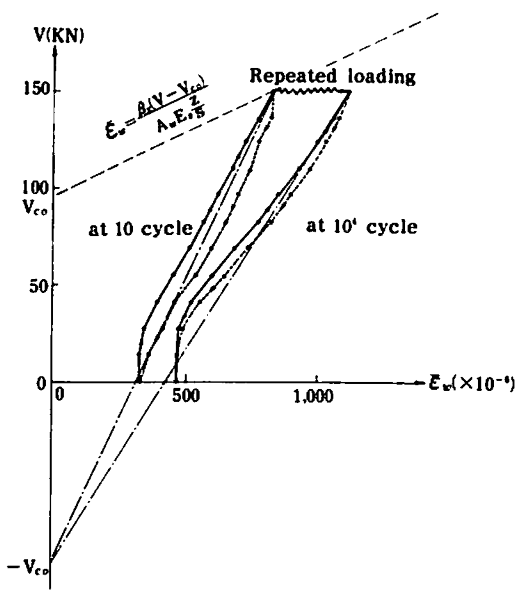


Fig.6 Obsreved and assumed relationships between average of stirrup strains and applied shear force (FS5)

It was observed that relationship between width of diagonal crack and applied shear force was very similar to that between the strain of stirrup in the vicinity of the diagonal crack and applied shear force. The width was closely proportional to the strain. This indicated the strains of stirrups corresponded to the propagation of diagonal cracks.

The observed $\bar{\varepsilon}_w$–V relationship in Fig.6 can be explained by assuming that $\bar{\varepsilon}_w$–V curve is always on the line between the point ($\bar{\varepsilon}_{wmax}$, V_{max}) representing the strain at the applied maximum shear force and the fixed point (0, $-V_{co}$). The strain range, $\bar{\varepsilon}_{wr}$, and residual strain, $\bar{\varepsilon}_{wo}$, are assumed to increase during fatigue loading, because the maximum strain, $\bar{\varepsilon}_{wmax}$, increases proportional to the logarithm of loading cycles according to Eq.(1). From this assumption, the following equations are derived.

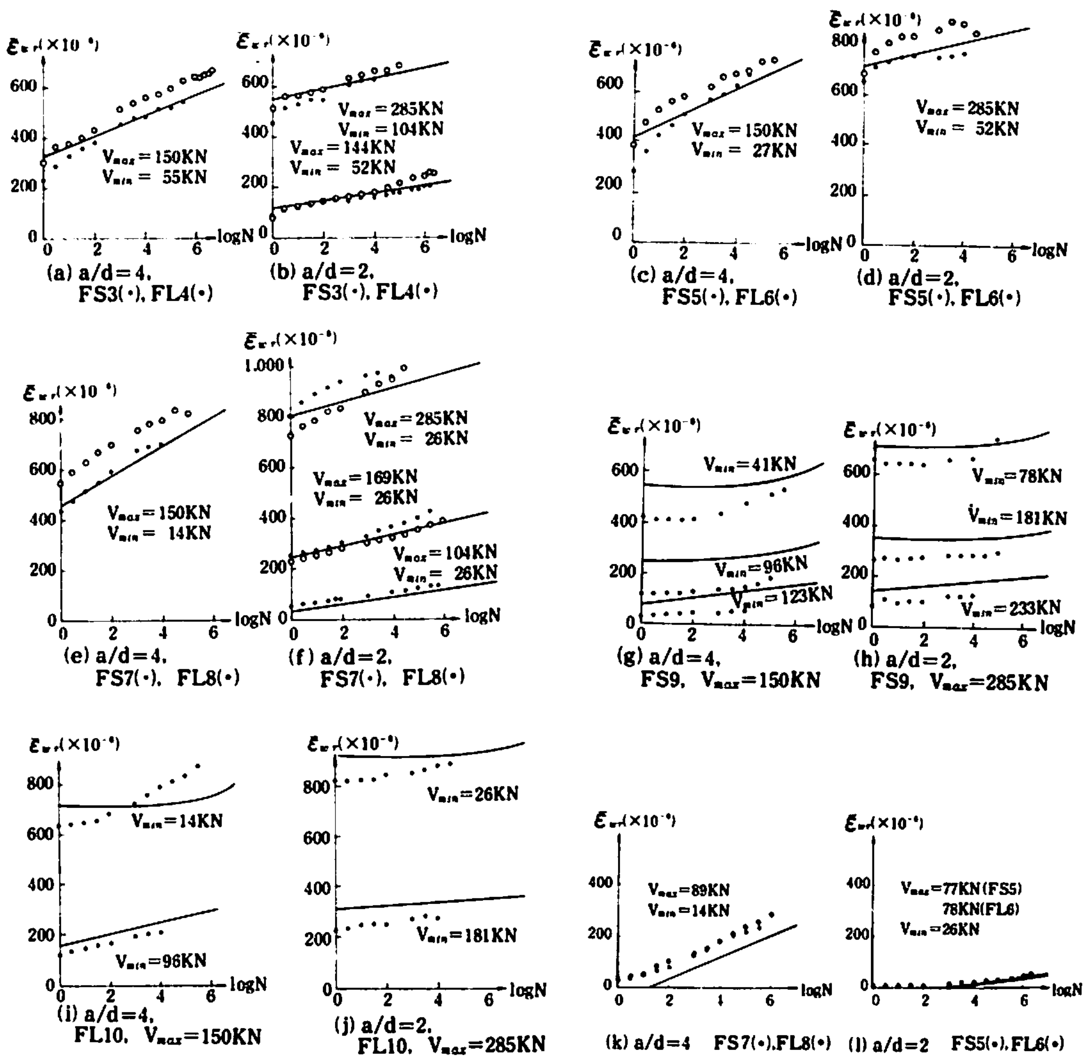
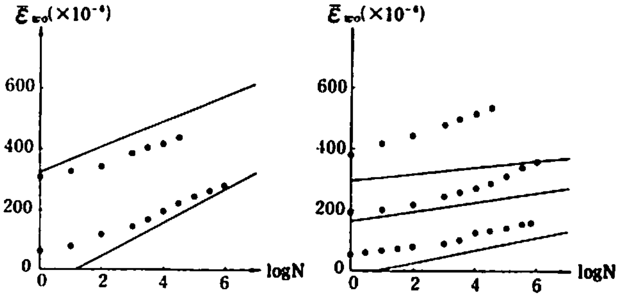


Fig.7 Average of strain ranges in stirrups

$$\bar{\varepsilon}_{wr} = \frac{V_{max} - V_{min}}{V_{max} + V_{co}} \bar{\varepsilon}_{wmax} \qquad (5)$$

$$\bar{\varepsilon}_{wo} = \frac{V_{co}}{V_{max} + V_{co}} \bar{\varepsilon}_{wmax} \qquad (6)$$

where $\bar{\varepsilon}_{wmax}$ is given by Eq.(1), and the influence of the knee in the $\bar{\varepsilon}_w$–V curve is neglected. The calculated values are compared with the tested values in Figs.7 and 8. The equation for the calculation of strain range of stirrup nicely expresses the average behavior of stirrup under fatigue loading.



(a) a/d=4 FL8(II)(III) (b) a/d=2 FL8(I)(II)(III)

Fig.8 Average of residual strains in stirrups

4. STIRRUP STRAIN UNDER FATIGUE LOADING WITH MULTI-LEVEL LOAD RANGE (STIRRUP STRAIN UNDER GENERAL VARIABLE LOADING)

Since actual structures are not subjected to fatigue loading with constant load range but generally subjected to fatigue loading with varied load range and/or sustained loading, it is necessary to follow the behavior of web reinforcement under fatigue loading with varied load range. Therefore, each specimen was subjected to fatigue loading with multi-level load range. Especially FS11 was subjected to sustained loading as well.

When general fatigue loading is divided into some sets of repeated loading with constant maximum shear force, each set is named the first repeated loading, the second repeated loading and so on according to the sequence of loading. Although the stirrup strain after subjected to the first repeated loading can be calculated as mentioned in Secs.2 and 3, the change of strain during the second repeated loading may not be calculated. But, this change can be estimated if it is assumed that loading history of the first repeated loading is equivalent to certain cycles (Neq) of repeated loading whose maximum shear force is equal to that of the second repeated loading. All the changes of stirrup strain under the subsequent repeated loaing can be estimated by the similar way.

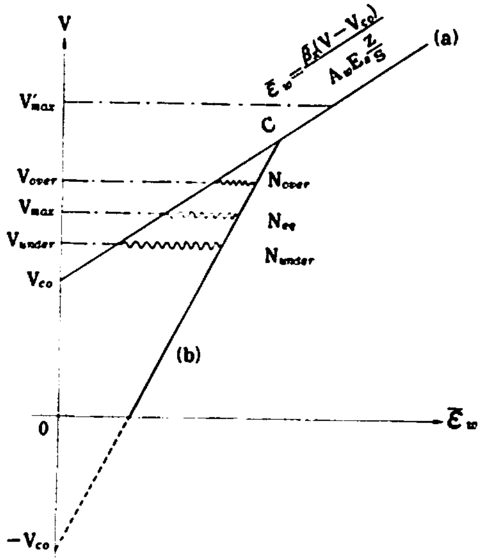


Fig.9 Idea of equivalent cycles

The line(b) in Fig.9 is drawn

between the fixed point (0, $-V_{co}$) and the point representing the strain of stirrup after subjected to N_{over} cycles of the first repeated loading whose maximum shear force is V_{over}. When the maximum shear force, V_{max}, of the second repeated loading is below the point C in Fig.9, the points representing the strains at the begining of the second repeated loading are on the line(b), as mentioned in sec.3. The strain, $\bar{\varepsilon}_w$, at V_{max} is calculated by the following equation.

$$\bar{\varepsilon}_w = \frac{V_{max} + V_{co}}{V_{over} + V_{co}} \bar{\varepsilon}_{wover} \qquad (7)$$

where $\bar{\varepsilon}_{wover}$ is the maximum stirrup strain calculated by using Eq.(1) with V_{max} = V_{over} and N = N_{over}. It is assumed that the state of strain after subjected to N_{over} cycles of the first repeated loading is equivalent to the state of strain after subjected to the equivalent cycles, N_{eq}, of loading whose maximum shear force is V_{max}.

A generalized statement of this assumption is as follows. If stirrup strains produced by the shear force applied are same in beams subjected to different loading historys, the changes of the strains during subsequent loading are essentially same in spite of the difference of the previous loading historys. In other words, the behavior of a stirrup after subjected to a certain loading, static or fatigue or sustained loading, is only dependent on the strain corresponding to the shear force applied. Consequently any loading history can be substituted by an equivalent fatigue loading with the constant maximum load.

This assumption is expressed by Eq.(8).

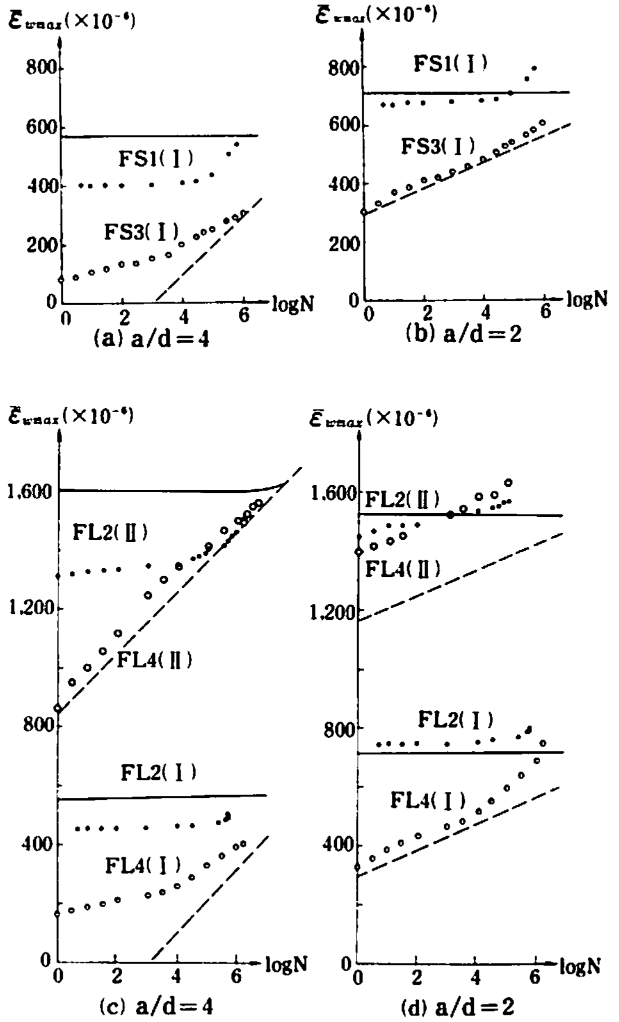


Fig.10 Influence of previous over-loading (The tested and calculated values in the beams without application of V_{over} are shown by open circles and dotted lines respectively.)

$$\frac{\bar{\beta}x\ \{\ Vmax - Vco\ (\ 1 - k\ logNeq\ )\ \}}{Aw\ Es\ z/s} = \frac{Vmax + Vco}{Vover + Vco}\ \frac{\bar{\beta}x\ \{\ Vover - Vco\ (\ 1 - k\ logNover\ )\ \}}{Aw\ Es\ z/s} \quad (8)$$

The equivalent cycles, Neq, can be obtained by transposing Eq.(8). When the maximum shear force, Vunder, of the first repeated loading is less than that of the second repeated loading, the equivalent cycles can be calculated by the similar way. After subjected to N cycles of the second repeated loading, stirrup strain, $\bar{\varepsilon}$wmax, at Vmax can be calculated by Eq.(1), substituting Neq+N for N. This means that the rate of increase in $\bar{\varepsilon}$wmax is very small if the figures of N is smaller than those of Neq. Solid circles in Fig.10 show the observed relationship between $\bar{\varepsilon}$wmax and logN in the cases of beams, in which Nover is one, and the solid lines in the figure are calculated by Eq.(1), substituting Neq+N for N.

When the maximum shear force, Vmax', of the second repeated loading is above the point C in Fig.9, the point representing the stirrup strain at Vmax' is on the line(a). The line(a) represents the relationship between stirrup strain and applied shear force under static loading. Therefore, there is no influence of the previous fatigue loading on the stirrup strains under the second repeated loading, and Eq.(1) can be used without any modification for calculating $\bar{\varepsilon}$wmax. Measured and calculated strains in this case are shown in Fig.3(a)(b)(c)(e)(f)(g).

The comparison between the calculated strains and the measured ones under fatigue loading of multi-level load range is shown in Fig.11. The data in Fig.11(a) taken from the specimen FS7 of the authors' test are the average strains of all the stirrups, but those in Fig.11(b) taken from Ruhnau's test [4] are stresses of one stirrup.

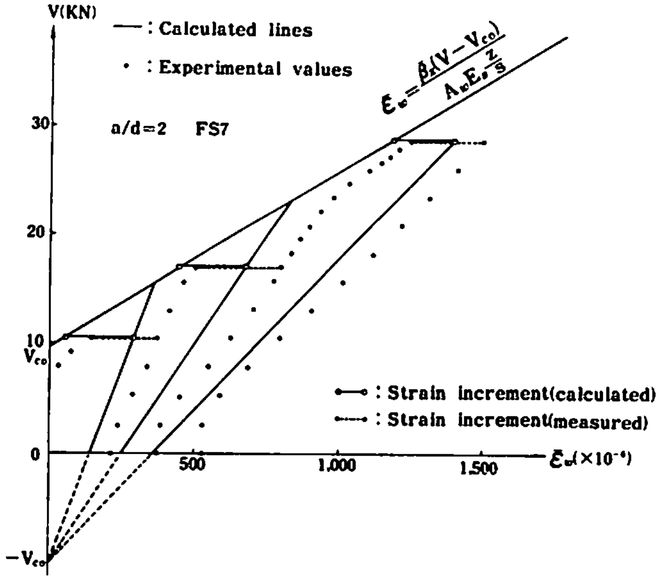
Stirrup strains increase clearly, whenever sustained load is applied. It was observed in specimen FS11 that the rate of increase was in proportion to logarithm of duration of the loading. When a stirrup, whose strain had increased due to sustained loading, was subjected to fatigue loading, it was observed that the increase of strain was small in the early stage of the fatigue loading. When sustained loading was applied after some cycles of fatigue loading, the increase of stirrup strain was hardly recognized. Therefore, the increase of strain due to sustained loading is considered essentially same as the one due to fatigue loading. Considering this fact, loading speed may be one of the important factors. But the effect of sustained loading, which may occur during a usual fatigue test may generally be negligible.

5. FATIGUE FRACTURE OF STIRRUP AND FATIGUE LIFE OF BEAM

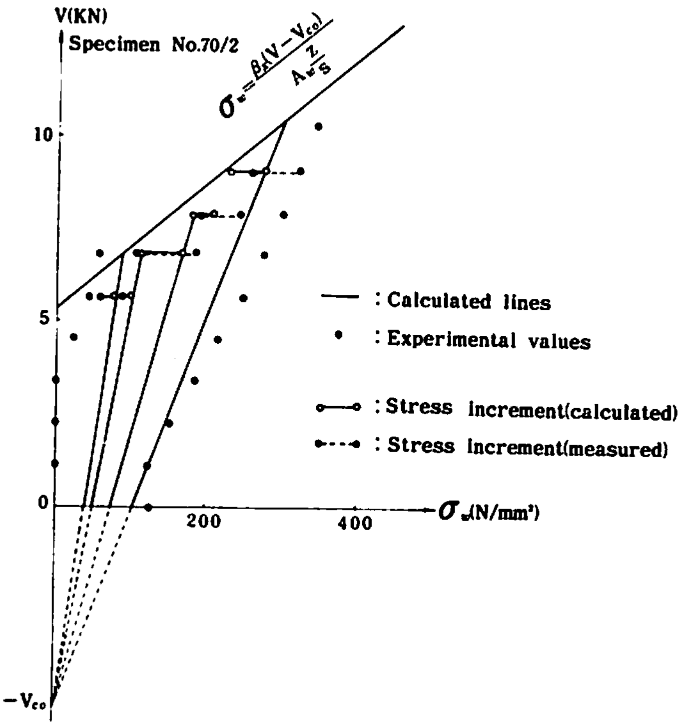
Fatigue fractured stirrups were always found in the shear span where a/d was 2.0, and none of fracture occurred in the shear span where a/d was 4.0. After the first fatigue fracture of stirrup, each specimen resisted two hundred thousand to three million cycles. Following the first fracture, specimens failed due to the fracture of four to ten more legs of stirrups. Although the total number of fractured stirrups in each specimen did not seem to relate to the magnitude of fatigue loading or the failure lives of the beams, it was recognized that the shorter the failure life of beam was, the fewer was the loading cycles after the first fracture.

The fatigue fracture occurred not only at lower bent portion where stirrup was bent around longitudinal bars but also at middle straight portion and upper hook portion. The portion of fatigue fracture was generally along the main diagonal crack which caused the failure of beam (see Fig.12). Many fractured legs at the middle straight portion of stirrup were found in the center of shear span, and those at the upper hook were found in the vicinity of the loading point. This fact is different from the previous test [1] in which all the fatigue fracture except for one occurred at the lower bend.

It is considered that the fatigue life of beam failing in shear due to stirrup fracture is related to the fatigue strength of stirrup. However, the measured strains shows no more information except for that the fatigue strength of stirrup lies between the fatigue strength of the straight bar and that of the bent bar. Because it is difficult not only to recognize the



(a) average strain of stirrups in FS7



(b) strain of a stirrup in No.70/2 [4]

Fig.11 Comparison between calculated and tested values of stirrup strains in the beams subjected to fatigue loading of multi-level

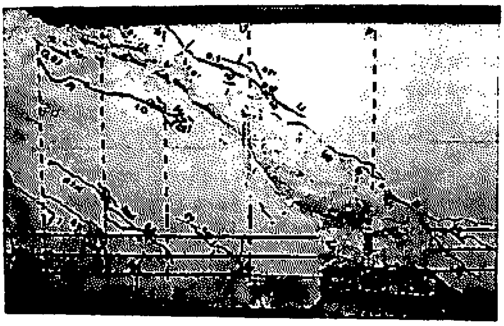


Fig.12 Examples of fatigue fractured stirrups (FL10)

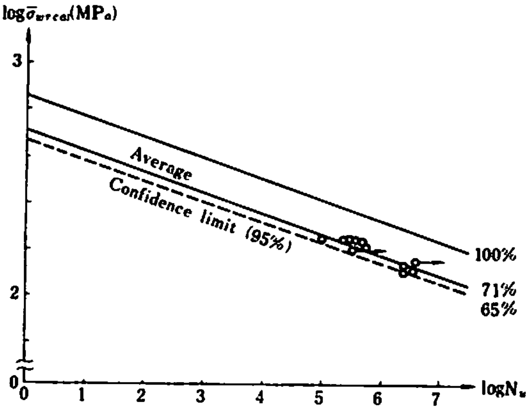


Fig.13 Relationship between calculated values of average stress ranges in stirrups at beam failure and tested ones of fatigue lives of beams

fracture from the measured strains, but also to clear the relationship between the strain at the measured point and the strain at the fractured point. On the other hand the closed relationship between calculated values of average stress ranges in stirrups at the the failure of beams and tested values of fatigue lives of beams is found as shown in Fig.13. The average of the stress ranges is 71% of the fatigue strength of the straight bar. This figure clears that the fatigue life of beam can be estimated from the average of stress range in stirrups calculated by using Eq.(5).

CONCLUSIONS

Nine of the eleven specimens failed in shear under fatigue loading, although these specimens under static loading would have failed in flexure at the load when stirrups yield. In all the cases shear failure occurred due to fracture of several legs of stirrups. The number of fractured legs in each specimen did not seem to be related to the failure life of the beam. It was recognized that the shorter the failure life was, the fewer was the subsequent loading cycles after the first fracture of a stirrup. About half of the fractured legs were found at the middle straight portion while all the fractured legs were found at the bent portion in the previous test [1]. It could be said that most of the fatigue fracture occurred along the main diagonal crack in the present test. As for the fatigue strength of stirrup, no further information is obtained except that it lies between the fatigue strength of the straight bar and that of the bent bar. However, it is made clear that the fatigue life of beam can be estimated from the calculated stress range in stirrup. Therefore, the investigation on stirrup strains under fatigue loading is important. Concerning the problem, following conclusions are obtained.

(1) It is proved experimentally that the previously proposed equation for the calculation of the average stirrup strain at the applied maximum load under fatigue loading does not depend on the shape of cross section nor on the shear span depth ratio, and is applicable not only to the cases of the

constant minimum load but also to those of the constant maximum load or of the constant load range. When applied maximum shear force is smaller than the shear capacity of concrete, the stirrup strain does not increase at the early stage of fatigue loading but begins to increase at the specific cycles of fatigue loading. The stirrup strain can be calculated also by the above equation where N is the total loading cycles from the start of the fatigue loading.

(2) An equation for the calculation of the average of strain ranges in stirrups under fatigue loading is newly proposed to increase the accuracy of the previous one. This equation is derived from such observation that the applied shear - stirrup strain relationship can be considered practically linear at unloading and reloading and the line representing this relationship is assumed to cross the shear axis at a fixed point.

(3) A procedure to calculate the average of strains in stirrups under general variable loading is proposed. This procedure is based on such a newly developed idea that the changes of the strains during the subsequent loading are essentially same in spite of the difference of the previous loading historys, when stirrup strains produced by the shear force applied are same in beams subjected to different loading historys. Consequently any loading history can be substituted by an equivalent fatigue loading with the constant maximum load.

AKNOWLEDGEMENT

The research was supported by the Grant-in-Aid for Scientific Research No.246118 from the Japanese Ministry of Education. Dr. Jun Yamazaki, Associate Professor of Tokyo Metropolitan University, gave valuable suggestions in English translation, and Mr. Matsuji Enomoto, Assistant of the University of Tokyo, helped in experimenting. The authors heartily express their gratitude for the warm and devoted co-operation.

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