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SHEAR DESIGN OF REINFORCED CONCRETE BEAMS FOR STATIC AND MOVING LOADS

By Hajime OKAMURA* *and Sabry* FARGHALY**

SYNOPSIS

The results of the test of a reinforced concrete T-beam 750 mm high and 7 500 mm long are presented. The beam was designed for a simulated moving load and was loaded until failure. The web reinforcement consists of vertical stirrups with variable intensity along the beam. A design rule for reinforced concrete beams for shear based on the yield of web reinforcement as one of the ultimate limit states is presented. Reduction for the design shear forces in the vicinity of concentrated loads and supports is duly recommended. Results obtained from tests on another T-beam and published test results are in good agreement with the presented design rule including the proposed reduction.

1. INTRODUCTION

(1) Scope

How much shear reinforcement is necessary and how such reinforcement should most effectively be used? This old question is still attracting attention to researchers on concrete structures. Because of complexities involved in the behaviour of concrete members under shear forces, the design rules for shear reinforcement differ considerably according to codes. However, most of them are based on so called truss analogy and are made by referring to test results of beams with constant spacings of stirrups.

The main object of this study is to propose a proper design rule for shear reinforcement based on a modified truss model. In the first place a large T-beam designed for a simulated moving load based on a trial design rule was tested. According to the difference in shear forces corresponding to loading positions, the spacings of vertical stirrups in the beam varied along the beam. From the analysis of the test, a new design rule for yielding of stirrups was obtained. In order to verify the rule, another large T-beam designed based on this rule was tested in the second place.

(2) Angle of compression diagonals

In the classical truss analogy, a beam is replaced by a statically determinate truss with parallel chords, and the angle of compression diagonals is assumed to be equal to 45°. However, recent works show that this angle can be lower, influenced by such parameters as the ratio of web to flange width, percentage of web reinforcement, presence of axial force and effect of loading point and support.

Stüttgart tests (1) show that the angle of diagonal crackings decreases with increase of the ratio of web to flange width. Thürlimann's analysis (2) indicates the proportional relation of the angle of compression diagonals with the percentage of shear reinforcement. The angle of diagonal crackings is affected by the application of axial compressive forces in an inversal relation. The crack patterns given in Ref. 3) and 4) indicate that the angle of diagonal cracking can be as low as 25°.

For full utilization of web reinforcement, angle of compression diagonals θ is required to be as low as possible. A limit of θ is given as 26.6° (cot $\theta=2$) by Thürlimann[2] and Regan & Placas[5]. This lower limit was used in the design of the first beam.

(3) Contribution of other means than the assumed truss

In the classical truss analogy all the applied shear force is to be carried by the assumed truss.

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* Assoc. Prof., Dept. of Civil Eng., University of Tokyo

** Graduate Student, Dept. of Civil Eng., University of Tokyo

However, recent tests show that the considerable part is carried by other means than the truss such as compression zone of concrete, dowel action of longitudinal bars and internal shear transfer at cracks. Nevertheless, contribution of these factors was neglected in the design of the first beam, since the angle of compression diagonals in the truss was taken as low as possible.

(4) Reduction of applied shear

In many works on beams without shear reinforcement, it was remarked that the shear strength became larger if the position of loading became nearer to the support. This was explained by arch action (9) or by local stresses around the support (10). In the truss model this can be treated as the inclination of the compression chord (1, 7, 8), and this produces considerable reduction in shear reinforcement near supports.

For design purpose this can be treated with reduction of applied shear forces near supports. The reduction was recommended to be applied for a concentrated load within the distance of 2d from supports (1), where d is the effective depth. Higai[11] found from his tests on specially reinforced beams that shear resistance of any part of the beam was affected by the position of loading, and it increased exceedingly if the load approached to the part within the distanse of 1.5d. He recommended the reduction of applied shear to half of its value at the loading point.

In order to take into consideration of these effects, applied shear forces in the design of the first beam were reduced in the vicinity of loading points and supports. The reduction began at the distance of 2d from supports or d from loading points, and changed linearly toward the support or the loading point. The reduction was 100% at the support and 50% at the loading point.

2. TEST OF THE FIRST BEAM

(1) Test specimen

In order to check the validity of the trial design rule mentioned before a T-beam designed for a simulated moving load was tested. The test specimen has 750 mm in height, 6600 mm in span and 7500 mm in length. The cross section is shown in Fig. 1.

Deformed bars were used throughout the specimen. The longitudinal tension reinforcement consisted of 16 bars with diameter of 25 mm. They were continuous through the beam length without any cut off. The web reinforcement consisted of vertical stirrups with nominal diameter of 13 mm. The measured yield strength was 35.5 kg/mm^2 and the modulus of elasticity was 2.17×10^4 kg/mm^2.

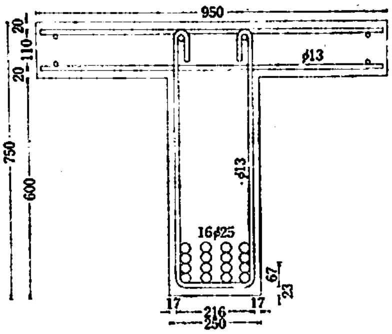

Fig. 1 Cross-sectional dimensions of the test beams (mm).

One batch of ready mixed concrete was used. It was made from a mixture of normal portland cement, sand and coarse aggregate with maximum size of 20 mm. The compressive strength determined from tests of control cylinders was 332 kg/cm^2. The test specimen and the control cylinders were cured in wet condition for 7 days, and then were left in the laboratory to dry until they were tested at 28 days.

(2) Design of stirrups and test procedure

The beam was designed for a load moving on the left half of the beam. The load consists of two indentical concentrated loads 600 mm apart moving with intervals of 600 mm from the position A to the position E as shown in Fig. 2. To obtain more information around the support, two additional loadings A′ and B′ with one point loading were also applied. The shear diagram for a certain load P modified by the reduction in the vicinity of supports and loading points is shown in Fig. 2.

The design load was 80 tons for the yielding of stirrups. According to the modified shear diagram and the difference in shear forces corresponding to loading positions, the intensity of designed stirrups varied along the beam. The calculated shear capacity of the beam at the positions of stirrups according to the trial design rule is also given in Fig. 2.

The beam was tested in a steel frame. Loads were applied by two hydraulic jacks with 100

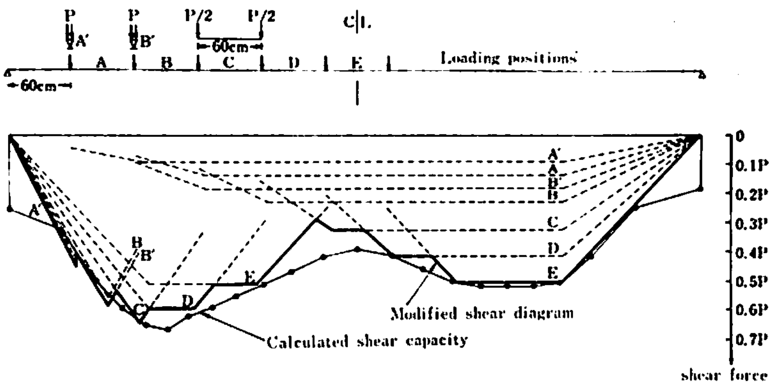

Fig. 2 Loading positions, modified shear diagrams for each loading position (A–E) and calculated shear capacity of the first beam at the positions of stirrups according to the trial rule.

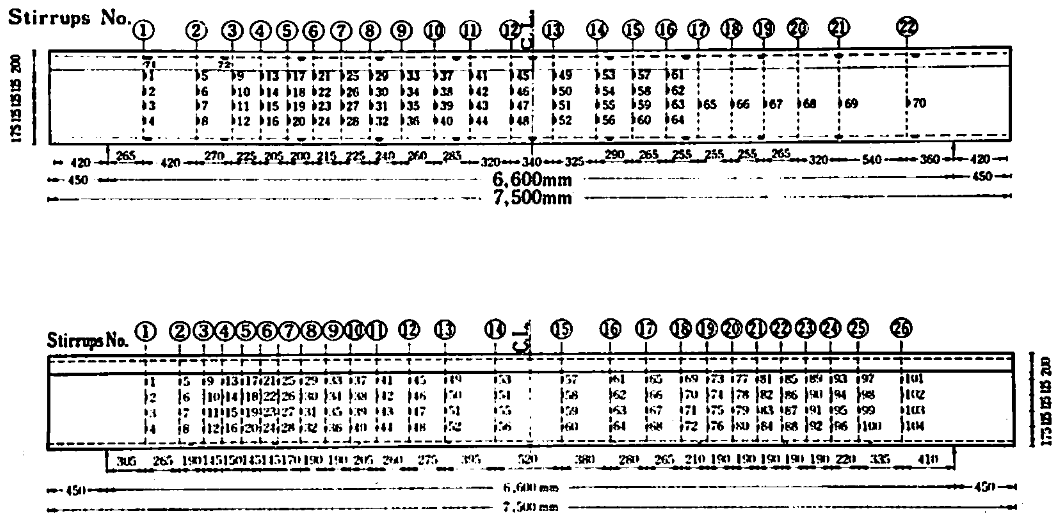

Fig. 3 Location of stirrups and positions of strain gauges. (a) the first beam, (b) the second beam.

ton capacity each. To distribute the loads, steel plates of 30×80×200 mm were used, and the beam was supported on roller bearings with 100 mm wide. Loading was applied in increments of 20, 40, 60, 80 and 90 tons at which the failure occurred. For every increment of loading the load was increased in three steps.

In order to take extensive measurement of strains on stirrups, all stirrups in the left part of the beam were instrumented with four strain gauges on the one leg of them. In the right part, which is mainly affected by the loading at position E, one strain gauge was provided on each stirrup. The positions of these gauges are shown in Fig. 3(a). Strain gauges were also provided on the bottom surfaces of longitudinal bars. The length of strain gauge was 10 mm.

3. TEST RESULTS AND DISCUSSIONS OF THE FIRST BEAM

Fig. 4(a) shows the crack pattern and the cracking loads with the position of loading. Most of diagonal cracks originated at the web between the longitudinal bars and several centimetres below the flange, independently of any

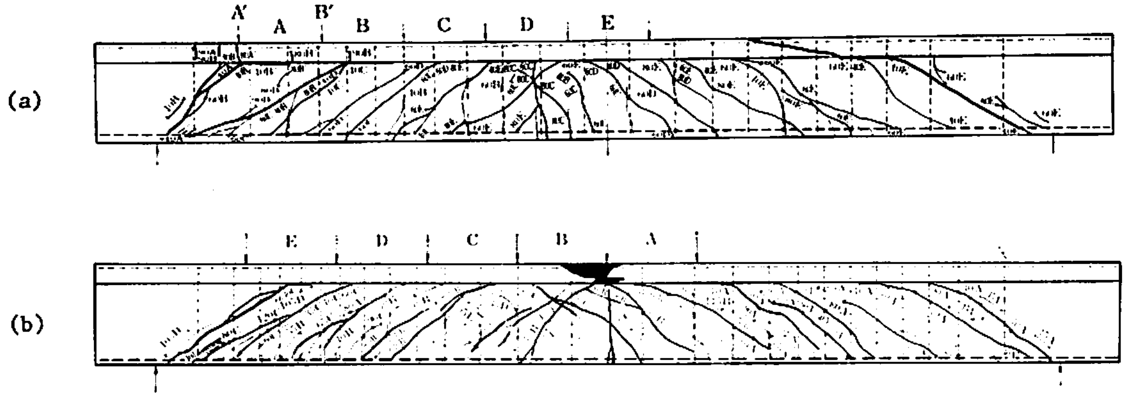

Fig. 4 Crack pattern and cracking loads. Numbers indicate the cracking loads in ton. The letters indicate the loading positions. (a) the first beam, (b) the second beam.

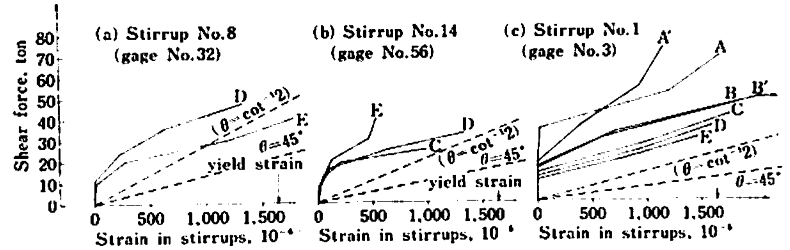

Fig. 5 Relationships between the applied shear forces and measured strains in selected stirrups of the first beam. For other loading positions not presented in (a) & (b) strains are negligible.

existing flexural cracks. The cracks were initially developed at about 45°. Further loading led the cracks to extend up or down to the full web depth. During the increase of loading additional flatter cracks branched from the formed cracks and became the main cracks. The angle of inclination of these main cracks ranged between 25° and 35° except those in the centre of the beam. From the inner edge of the left support two inclined cracks with 45° and 25° were created.

Soon after a diagonal crack crossed the stirrup, the strain at the position near to the crossing point began to increase and continued to increase until yielding in approximately linear proportion with the applied shear as shown in Fig. 5 for the selected stirrups. The positions away from the crossing of the crack did not show such increase of strain as in Fig. 5. For more details Table 1 gives all values of measured stirrup strains for all measurement positions under all loading positions except four positions at which their strain gauges were damaged.

The rate of increase in the strains of most stirrups was considered to be parallel to the line obtained from assuming the truss with 45° diagonals rather than that with $\cot^{-1}2$, which was assumed in the design. Examples are shown in Fig. 5(a) and Fig. 5(b).

Four stirrups, No. 1, 2, 21 and 22, which were adjacent to the supports showed smaller rate of increase in strain. A typical example is given in Fig. 5(c). This behaviour was expected in the design, but the reduction of applied shear in the vicinity of supports seemed better to be applied not to the total shear but to the strains of stirrups only. This is discussed later. It was also recognized that the rate of increase in the strain became smaller when the loading points were near to the stirrup. The strains at the position D in Fig. 5(a), E in Fig. 5(b) and A' in Fig. 5(c) are the examples.

From these measurement it could be considered

Table 1 The measured strains on stirrups of the first beam.

Stirrup No.	Load at: P, ton / Str. gage No.	A			A′			B			B′			C			D			E		
		40	60	80	40	60	80	40	60	80	40	60	80	40	60	80	40	60	80	40	60	80
1	1	−15	−269	−666	−5	−257	−552	−42	−620	−824	−62	−747	−814	−266	−740	−780	−235	−726	−738	−233	−715	−701
	2	−15	+586	+994	+283	+490	+727	+234	+1037	+1247	+312	+1196	+1258	+476	+1091	+1137	+470	+1020	+969	+436	+961	+844
	2	+25	1205	+1661	+407	+925	+1134	+545	+1826	+6199	+707	+2039	****	+972	−1761	****	+912	+1599	****	+789	+1460	****
	4	−38	+446	+1142	+21	+299	+853	+67	+1023	+1432	+242	+1303	+2018	+344	+1199	+1438	+344	+1120	+1743	+322	+1050	+1611
2	5	−48	+298	+367	−96	−108	−173	+123	+1466	+2050	+703	+1720	+2441	+758	+1417	+3891	+698	+1238	+3679	+596	+1094	+3443
	6	−49	+157	+340	−64	+65	+128	+109	+362	+1047	+163	+483	+1124	+195	+526	+1336	+198	+551	+1303	+189	+541	+1231
	7	−20	+47	+500	−26	+11	+177	+27	+154	+1445	+49	+294	+1422	+55	+631	+1634	+59	+665	+1620	+56	+657	+1428
	8	+62	+163	+519	+33	+91	+137	+109	+418	+1819	+135	+497	+1740	+139	+847	+6707	+154	+893	+8190	+143	+832	+7449
3	9	−2	+36	+259	+1	+43	+216	−3	+195	+861	+37	+231	+1000	+106	+368	+1133	+106	+426	+1120	+99	+429	+1050
	10	−1	+179	+480	+3	+141	+247	+15	+450	+1921	+56	+385	+1480	+196	+994	+1356	+263	+963	+1307	+249	+922	+1083
	11	+58	+400	+659	+37	+151	+311	+273	+940	+1974	+348	+771	+1249	+589	+1931	+2128	+637	+1856	+1907	+558	+1693	+1690
	12	−32	+137	−484	−26	+97	+214	−4	+432	+1134	+23	+391	+872	+146	+1113	+1309	+220	+1384	+1192	+211	+1016	+1081
4	13	−72	−248	−268	−57	−353	−380	−36	+638	+1007	−22	−265	−317	−380	+1460	+2167	+646	+1236	+2011	+636	+1155	+1809
	14	−11	+89	+170	+15	+112	+207	+71	+313	+988	+27	+89	+149	+354	+1437	+2033	+362	+1256	+1832	+327	+1146	+1612
	15	−58	+11	+160	+29	+32	+163	+37	+107	+455	+42	+19	+143	+88	+425	+785	+110	+519	+851	+101	+510	+823
	16	+6	+23	+171	+4	+35	+152	+14	+83	+573	−3	+44	+171	+63	+336	+1086	+66	+787	+1346	+66	+757	+1227
5	17	−6	−89	−87	−13	−139	−199	−5	−249	+462	−25	−57	−57	−334	+1165	+2856	+443	+939	+2461	+402	+860	+2260
	19	+8	+51	+208	+13	+58	+215	+53	+149	+643	+16	+63	+167	+125	+260	+1488	+122	+1176	+2238	+113	+1168	+2078
	20	+13	+56	+186	+22	+67	+193	−1	+43	+523	+24	+87	+159	+14	+368	+1200	+21	+916	+1577	+26	+911	+1439
6	21	+6	+26	+253	+6	+38	+262	−12	+24	+269	−2	+21	+200	+54	+163	+755	+89	+451	+1211	+81	+523	+1380
	22	−3	−4	−164	−6	−20	−193	−10	−27	−326	−6	−9	−118	−56	−78	+1118	−59	+972	+2101	−62	+1034	+1977
	23	+3	+31	+159	+4	+36	+173	+19	+58	+321	+13	+38	+110	+51	+402	+1001	+68	+1009	+1686	+66	+1004	+1577
	24	+36	+97	+194	+35	+96	+204	+24	+140	+380	+47	+115	+156	+67	+669	+1159	+122	+1231	+1948	+144	+1173	+1808
7	26	+15	+63	+197	+10	+63	+221	−30	−11	+92	+24	+65	+121	−7	+382	+740	+90	+1271	+1810	+87	+1163	+1727
	27	+14	+29	+143	+13	+43	+151	−11	+55	+112	−13	+26	+83	+2	+292	+460	+204	+624	+1013	+201	+624	+1054
	28	+12	+85	+190	+12	+85	+177	+8	+77	+163	+37	+111	+189	+3	+104	+365	+44	+328	+792	+83	+594	+1082
8	29	−9	−7	+202	−7	−12	+185	−8	−8	+85	−10	+16	+144	−6	−29	+175	−15	+255	+899	−2	+599	+983
	30	+9	+11	+215	+8	+19	+214	−1	+4	+144	+13	+23	+160	+20	+9	+288	−10	+146	+580	+11	+500	+976
	31	+45	−130	+179	+39	+126	+184	+60	+146	+155	+66	+149	+182	+18	+170	+325	+90	+501	+1117	+180	+1017	+1782
	32	+7	+48	+151	+10	+51	+162	−3	+42	+99	0	+38	+113	+56	+230	+419	+215	+634	+1313	+275	+1097	+1769
9	33	−12	−62	−153	−9	−61	−159	−15	−58	−89	−11	−51	−108	−7	−49	−134	0	+443	+101	−161	+925	+1282
	34	+29	+83	+169	+24	+80	+172	+28	+70	+114	+26	+65	+120	+47	+185	+218	+259	+522	+725	+350	+941	+1309
	35	−1	−24	+155	−2	+35	+157	−9	−6	+106	−9	−15	+103	−39	−20	+148	+30	+179	+437	+88	+477	+638
	36	+14	+48	+147	+12	+56	+144	+31	+85	+174	+30	+81	+177	−2	+37	−122	−11	+31	+331	+5	+395	+762
10	37	+16	+21	+127	+9	+22	+122	+17	+23	+99	+12	+22	+102	−15	−23	+33	+24	+73	+129	+84	+311	+500
	38	−17	−33	−191	−13	−39	−196	−13	−24	−162	−13	−25	−171	−24	−21	−147	−36	−144	−370	−134	−498	+859
	39	−11	−14	−186	−11	−1	−193	−28	−33	−179	−24	−22	−191	−60	−56	−190	−29	−4	−510	−37	+867	+1359
	40	−51	−222	+1595	−46	−181	+2501	−75	−211	+4783	−78	−205	+1223	−110	−217	−490	−56	−124	+1590	−50	+707	+1290
11	41	+17	+31	+132	+11	+31	+136	+18	+39	+144	+17	+36	+144	+17	+30	+292	+15	+57	+205	+68	+422	+892
	42	+27	+32	+160	+20	+36	+155	+26	+36	+206	+24	+36	+200	+26	+55	+471	+18	+53	+267	+73	+616	+946
	43	−17	−66	−238	−19	−71	−218	−22	−82	−329	−30	−88	−306	−59	−207	+624	−14	−73	−266	−19	−426	−602
	44	+49	+100	+162	+44	+92	+141	+67	+133	+193	+70	+123	+174	+101	+164	+276	+72	+109	+217	+38	+109	+283
12	45	−20	−26	−137	−13	−27	−113	−24	−41	−262	−23	−16	−236	−31	−46	−537	0	−145	−396	−2	−29	−13
	46	−25	−40	−115	−20	−44	−130	−26	−56	−164	−28	−58	−152	−36	−98	−232	−11	−103	−176	−39	−97	−103
	47	+49	+117	+206	+38	+107	+172	+73	+189	+295	+69	+175	+262	+142	+254	+422	+126	+253	+353	+25	+114	+123
	48	+1	+57	+116	+4	+67	+114	−13	+58	+142	−2	+62	+140	−5	+92	+382	+3	+86	+234	+63	+140	+169
13	49	−9	0	−170	−5	−7	−151	−6	−1	−222	−8	−1	−223	−7	−10	−632	−13	−171	−870	−34	−93	−168
	50	−1	−2	+359	−2	−26	+291	−1	−18	+549	0	−23	+497	0	−54	+906	−14	+532	+1171	−22	−208	−206
	51	−20	−73	−561	−19	−74	−396	−24	−94	+940	−26	−96	+775	−40	−294	+1373	−87	+974	+1546	−43	−300	−329
	52	−5	−70	−260	−5	−63	−204	−5	−110	−426	−6	−102	−366	−33	−235	+652	+100	−528	+781	−43	−211	−233
14	53	−12	−27	−143	−8	−28	−138	−16	−28	−160	−15	−30	−158	−18	−36	−192	−13	−106	−434	−8	−123	−513
	54	−1	−27	−215	−1	−32	−193	−8	−28	+281	−4	−33	+264	−4	−38	−401	−4	+287	+691	−9	+273	+504
	55	−20	+24	+348	−16	+30	+241	−36	+36	+579	−27	+41	+480	−35	+71	+896	+5	+582	+1045	+17	+511	+435
	56	+31	+75	+340	+27	+61	+231	+31	+116	+572	+34	+106	+474	+56	+195	+1004	+132	+667	+1338	+119	+461	+533
15	57	+9	+1	+148	+5	+7	+138	+6	−1	+170	+8	+6	+166	+4	+2	+197	−13	+42	+369	−21	+147	+678
	58	+5	+60	+233	+4	+65	+204	+3	+75	+304	+4	+76	+276	+2	+112	+375	+5	+206	+657	+67	+361	+1151
	59	+2	+144	+328	+6	+155	+293	−3	+213	+507	+5	+205	+433	−9	+294	+707	+61	+479	+1323	+193	+661	+1469
	60	+32	+113	+251	+27	+89	+193	+40	+175	+406	+41	+156	+347	+74	+250	+686	−148	+472	+1146	+210	+491	+1171
16	61	−10	−49	+131	−8	−53	+56	−11	−16	+300	−14	−25	+216	−14	+30	−482	−11	+352	+643	−146	+575	+747
	62	+2	+22	+103	+4	+27	+91	−1	+34	+139	+6	+35	+125	−1	+48	−177	+10	+96	+337	+37	+164	+742
	64	+13	+84	+216	+15	+77	+173	+10	+120	+321	+19	+116	+268	+807	+167	+952	+864	+746	+840	+136	+1200	+1503
17	65	+13	+206	+353	+14	+187	+322	+14	+288	+446	+18	+268	+403	+26	+176	+544	+132	+520	+685	+331	+597	+915
19	67	+3	+22	+287	+3	+24	+252	−1	+34	+372	+2	+31	+337	+1	+43	+459	+6	+83	+889	+30	+441	+1280
20	68	+7	+154	+323	+3	+141	+292	+1	+203	+401	+2	+190	+365	+0	+260	+488	+21	+368	+603	+219	+508	+1065
21	69	+13	+200	+661	+11	+185	+509	+8	+262	+898	+11	+250	+796	+10	+351	+1096	+13	+704	+1316	+109	+1154	+1400
22	70	+8	+42	+403	+7	+41	+383	+1	+51	+458	+11	+54	+434	+2	+68	+513	+4	+160	+609	+38	+507	+1000

(+)=tension, (−)=compression, (****)=already yielded.

that the applied shear was carried not only by the assumed truss but also by the other means. The shear V_c, which was carried by the other means than the truss, could be obtained by continuing the measured load-strain line toward the load axis. Measurement of V_c for each stirrup showed a considerable scatter as usually found in such measurement. In fact, it was found that for one stirrup crossed by two diagonal cracks with different inclination there were two different valves of V_c. However, the valves of V_c for the stirrups near supports were similar to those for others. This indicates that the reduction of applied shear does not seem to be applied to the valves of V_c. The average of V_c for the total measurement was 18.1 tons.

Under the design load of 80 tons six stirrups from the left support had yielded, but the beam could sustain the load of 90 tons at loading positions of A, A′ and B. At the next position B′ the carrying ability of the beam reached the maximum of 88.3 tons. After this maximum load was reached there was a substantial drop in load carrying capacity accompanied by accelerated vertical increase in the width of two V-shaped cracks at the left support specially of the steeper one as shown in Fig. 6. All the stirrups crossed by these cracks had yielded prior to this stage. Although it could be considered that the beam failed at this stage, the beam was reloaded at the position E. At this central loading position the load carrying capacity reached 106 tons at which failure occurred at the right side of the beam as shown in Fig. 4(a).

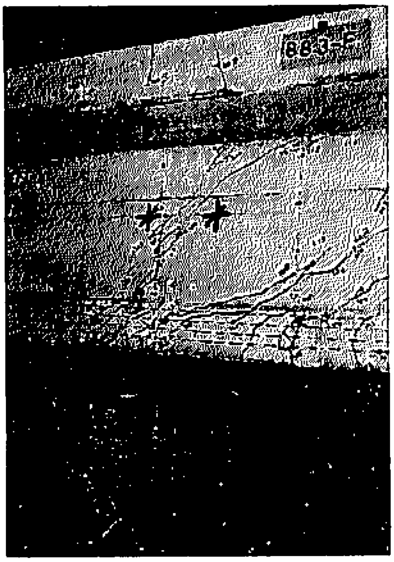

Fig. 6 First failure of the first beam.

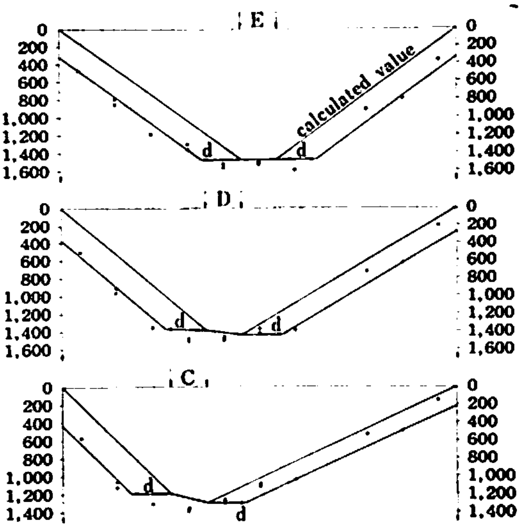

Fig. 7 The measured strains (10^{-6}) on the tension reinforcement of the first beam under the design load of 80 tons.

The measured strains on the tension reinforcement under the loading positions of C, D and E are given in Fig. 7. From this figure, a shifted length of this beam is suggested to be about the effective depth.

4. MODIFIED DESIGN RULE

(1) Shear carried by the assumed truss

From the general behaviour of the recorded strains on stirrups the shear force corresponding to the yielding of the stirrup V_y can be expressed as the sum of the shear force carried by the assumed truss with 45° diagonals V_{sy} and the shear force carried by other means V_c than the truss. Yielding of stirrups does not always mean the failure of beam, but it often occurs prior to the complete failure. However, under the yielding of web reinforcement, widths of diagonal cracks become abnormally wide and the deflection increases rapidly. Therefore, the yield limit state of web reinforcement is considered to be necessary in addition to the ultimate shear capacity.

$$V_y = V_c + V_{sy} \cdots\cdots\cdots\cdots (1)$$

$$V_{sy} = A_{sw} f_{wy}\, z/s \text{ for vertical stirrups} \cdots (2)$$

where,

A_{sw}=area of a stirrup
f_{wy}=yield strength of the stirrup
z=distance from the centroid of compression of concrete to the tension reinforcement
s=spacing of stirrups

For the design purpose, the value of V_c can be assumed at the present as the shear force corresponding to the opening of diagonal cracks and to be constant along the whole beam, although it is recognized to increase slightly near the support.

(2) Reduction of shear applied to the truss

For the design purpose, the reduction of applied shear due to the effect of supports and loads can be applied to only the truss part, keeping the value of V_c constant along the beam. In Fig. 8, the test results are plotted according to this principle. In other words, the shear forces V_{cal} carried by each stirrup position were calculated by Eq. (3) and the ratios of V_{cal}/V were plotted, where V is the applied shear force.

$$V_{cal}=V_c+V_s \quad \cdots\cdots (3)$$

where,

$$V_s=A_{sw}E_s\epsilon_w z/s \quad \cdots\cdots (4)$$

E_s=modulus of elasticity of stirrups $=2.17\times10^4$ kg/mm^2
ϵ_w=strain of the stirrup
V_c=18.1 ton for this case

The load P in Fig. 8 is the maximum applied load under which any stirrups in the same figure do not yield. The ratios of V_c/V vary according to the applied loads and the loading positions. Nevertheless, the ratios of V_{cal}/V decrease approximately linearly toward the corresponding values of V_c/V at the supports or the loading positions, and the reduction of applied shear force can be seen in the range of 1.5d from the supports and d from the loads. It is of special interest to note that the sum of these lengths is 2.5d, which is the critical length where the increase of shear strength of beams without web reinforcement starts. From the general trend of these results the reduction of applied shear force to the truss part is suggested as shown in Fig. 9. This reduction can only be applied where the

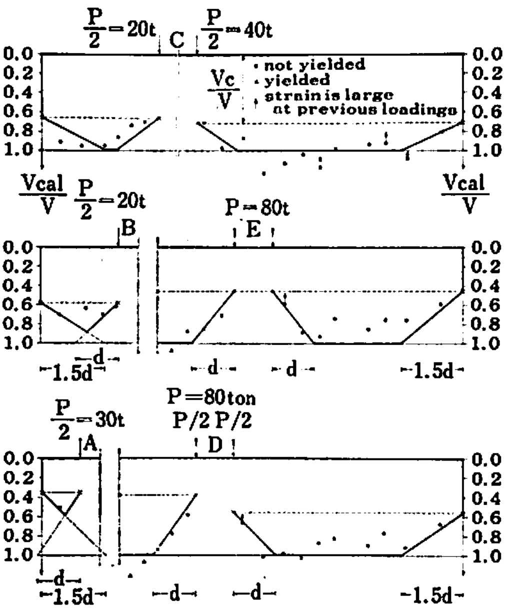

Fig. 8 Relationships between position of stirrup and the ratio of V_{cal}/V of the first beam, where, V is applied shear force and V_{cal} is shear force calculated by Eq. (3) using measured strain.

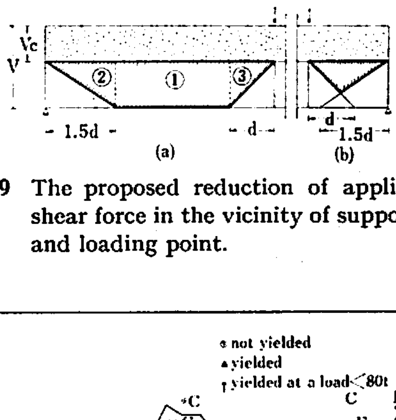

Fig. 9 The proposed reduction of applied shear force in the vicinity of support and loading point.

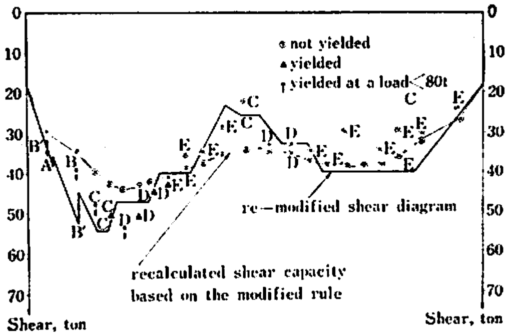

Fig. 10 The re-modified shear diagram, the calculated shear force of the first beam carried by each stirrup position with the symbol indicating the critical loading position, and the recalculated shear capacity based on the modified rule.

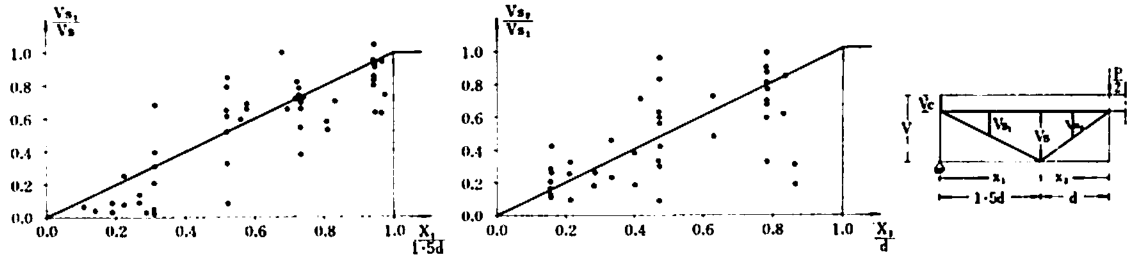

Fig. 11 Comparison of test results reported in literature with the proposed reductions.

supports or the loads produce transverse compressive stresses in the beam.

Using this reduction re-modified critical applied shear diagram was obtained as shown in Fig. 10. Although the shape of this diagram is different from the previous one in Fig. 2, the critical loading positions are similar. In Fig. 10 the calculated shear force of the beam carried by each stirrup position are also given together with the symbol indicating the critical loading position under which the strain of the stirrup took the maximum value. The critical loading positions obtained from the measurement are recognized to be consistent with the critical loading positions of the re-modified shear diagram, although some values in the right part of the beam are smaller than the diagram. In the right part, only one strain gauge was provided on each stirrup. This seems to make it difficult to pick up the maximum strain along the stirrup.

The shear capacities of each stirrup position were recalculated by Eq. (1).

The results are also shown in Fig. 10, which indicates that the yielding of stirrups occurred at the positions where the recalculated shear capacities are below the re-modified shear diagram.

(3) Application of the proposed design rule to available test data

To demonstrate the general applicability of the proposed design rule, available test results in published papers were reanalysed. Unfortunately, most of shear tests contain only the limited record of strain measurement. Strain measurement of stirrups is given for 15 beams in Ref. 12), and for 3 beams in Ref. 13). These eighteen beams were provided with stirrups running with constant intensity in each beam. The shear forces carried by stirrups V_s, V_{s1} or V_{s2} were calculated by Eq. (4). The results are presented in Fig. 11, which indicates that the ratios of V_{s1}/V_s and V_{s2}/V_s decrease toward the support and loading point, respectively. And the range of reduction can be considered about 1.5 d and d for the vicinity of the support and loading point, respectively, although the variation of data is rather large.

5. THE SECOND BEAM

(1) Design and test procedure

Analysing the results of the first beam, a modified design rule was proposed for the yield limit state of web reinforcement. To verify the applicability of this rule to the beam which is designed according to this rule, another T-beam was tested. This beam was designed for the similar simulated moving load starting from the position A to the position E in Fig. 12. The load consists of two identical concentrated loads 660 mm apart moving with intervals of 660 mm, which is equal to the effective depth. In this test the loading order and positions were not the same as in the first beam. The design load for the yielding of stirrups was also 80 tons. The applied shear diagram for the design load including the proposed reduction is shown in Fig. 12.

The second beam was identical with the first beam in shape, length and longitudinal reinforcement except in the amount of stirrups. The yield strength of stirrups was 33.1 kg/mm^2 with a strain of 2.1×10^{-3}. Spacing of the stirrups with a nominal diameter of 13 mm was designed for the critical shear diagram in Fig. 12 by using Eq. (1), taking V_c equal to the value in the first beam. The positions of stirrups are shown in Fig. 3(b) together with the gauge numbers. Strain gauges were mounted on four positions of one leg of each stirrup. The concrete strength was 355 kg/cm^2.

The second beam was tested in the similar manner as in the first beam. Loading was applied initially in increments of 10 tons and then the

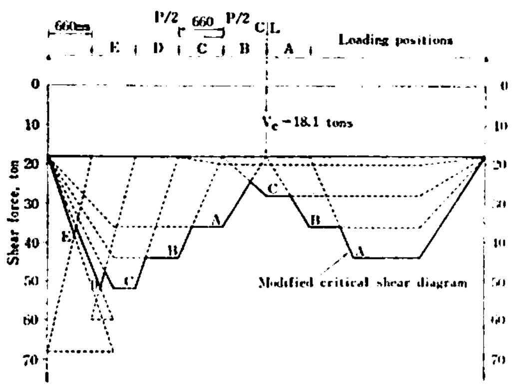

Fig. 12 Loading positions and modified shear diagrams for each loading position of the second beam.

increments were reduced to 5 tons after the diagonal cracks developed.

(2) Test results and discussion

Fig. 4(b) shows the cracks in the second beam, which indicated the similar cracking pattern to that in the first beam. The inclination of main cracks ranged between 31° and 35° in general, although the cracks initially formed were about 45° except the nearest crack to each support. The shear force V_{cr} at which the inclined cracks developed was assessed by visual observation and by the measurement of stirrup strains. All of the initially formed inclined cracks with one exception appeared when the load was increased from 30 to 40 tons at the loading positions of A, B and C, and the average value of V_{cr} was 19.25 tons. The cracking load of the fourth crack from the right support was the exception and was 80 tons ($V_{cr}=44$ tons).

Fig. 13 shows the shear forces carried by each stirrup position under the design load of 80 tons, which were calculated by Eqs. (3) and (4) together with the critical loading positions obtained. The strain of a stirrup in Eq. (4) is the maximum value of the four measurment on the stirrup. The details of the measurment can be given by request from the author. The modified shear diagram is also given in the figure. This figure clearly indicates that the shear forces carried by each stirrup position are consistent with the critical modified shear diagram. Thus, the proposed design rule is considered to be verified by the results of this beam designed according to the rule.

After the yielding of some stirrups, the beam could sustain the additional load, although the width of diagonal cracks became very wide. Under the load of 95 tons with the loading position of D, the first diagonal crack from the left support opened wide to some millimetres. By this loading condition twenty two stirrups out of 26 had yielded. The beam carried the load of 110 tons for all the loading positions. When the load was increased to 115 tons at the first loading position of A, the deflection of the beam increased largely. Therefore, the load was increased to 120 tons for the next position of B and then to C at which the first failure occured. At this loading position of C, a concentrated crushing of the lower tip of the triangular was observed under the load of 120 tons as shown in Fig. 14. At the next loading position of D the beam could carry only 109 tons, under which the same local crushing was observed and the first crack from the left support propagated into the full flange depth of one side in a very flat inclination toward the nearest loading point. It can be pointed out that the loads of 120 tons for C and 109 tons for D produce approximately same values of shear forces to this region. The beam was loaded with the loading position of A and carried 122 tons. Under this load the deflection increased rapidly, and accompanied with the local crushing

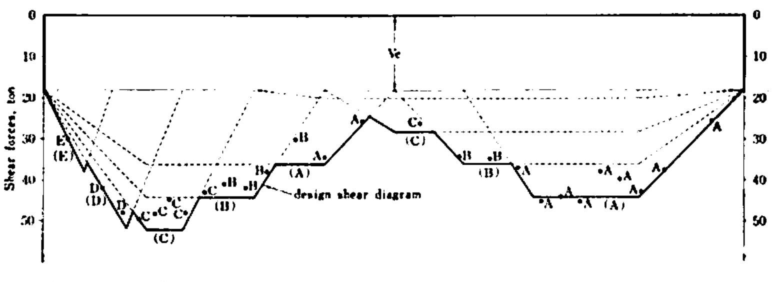

Fig. 13 Shear forces carried by each stirrup position of the second beam under the design load of 80 tons, which were calculated by Eq. (3) using the measured strains.

Fig. 14 Close-up view of failure in the second beam under the load of 120 tons at position C (the opposite surface of Fig. 4 (b)).

of concrete at the upper level of the longitudinal tension bars between the nearest two diagonal cracks to the right support. At the next loading position of B the beam finally showed a pure flexural failure at the load of 123 tons, accompanied with the distruction of compression zone.

6. CONCLUSIONS

(1) From the test results of a T-beam designed for simulated moving load, the shear capacity corresponding to the yielding of stirrups can be expressed as the sum of the shear carried by the assumed truss with 45 diagonals and the shear carried by other means than the truss.

(2) For the limit state of yielding of stirrups, the reduction of applied shear force in the vicinity of the supports and concentrated loads can be applied to only the truss part, keeping the shear carried by other means than the truss to be constant along the whole beam. The range of the reduction for T-beams shall be 1.5d from the supports and d from the loads.

(3) This proposed design rule for the limit state of yielding of stirrups was verified by the test results of the beam designed according to this rule.

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