(4)

TITLE NO. 67-57

Authorized reprint from copyrighted
Journal of the American Concrete Institute
November 1970, Volume 67, p. 894-897

AN ACI DIGEST PAPER

Research report sponsored by the
Reinforced Concrete Research Council

Elastic Frame Analysis—Corrections Necessary for Design of Short Concrete Columns in Braced Frames

By HAJIME OKAMURA, SHRINIWAS N. PAGAY, JOHN E. BREEN and PHIL M. FERGUSON

Ordinary frame analysis based on gross concrete sections generally leads to computed column moments and eccentricities at ultimate considerably in error (generally on the unsafe side) because of (1) the lower stiffness of the cracked beam, (2) the loss of stiffness of the column concrete near ultimate, and (3) sometimes the yielding of column compression steel on one face.

Short columns in single curvature in braced frames, analyzed by a computer procedure, show indicated eccentricities ranging from 0.52 to 2.08 times the nominal values obtained from ordinary moment distribution.

Moment distribution procedures based on radical changes in calculated member stiffness are indicated as feasible and empirical corrections are also presented for columns in braced frames.

Keywords: bending moments; columns (supports); eccentricity; frame analysis; reinforced concrete; short columns; structural design.

■ ELASTIC METHODS OF FRAME ANALYSIS are approximate in several ways when applied to reinforced concrete frames. The varying reinforcement and variable degree of cracking make the effective moment of inertia uncertain. Usually the gross moment of inertia of the concrete without an allowance for the steel is used.

While beams will typically be cracked, the columns often remain uncracked. The reduced stiffness of cracked beams thus leads to a larger distributed moment to the column. Because column moment is wholly determined by this distribution, the percentage difference is often large and designs would be more realistic if this excess initial eccentricity were considered.

Near ultimate the column concrete approaches its ultimate strength in compression and its effective modulus and stiffness are greatly reduced. If the compression steel strain reaches its yield zone, the reduction in concrete stiffness is accompanied by complete loss of some steel stiffness. The proportion of the beam moment distributed to the column at this stage decreases and the real eccentricity moves back towards the nominal value (elastic analysis), the degree of recovery depending upon the mix of the variables.

This loss of column stiffness may be small or it may entirely cancel the increase in initial eccentricity. Indeed, particular combinations of columns and beams occasionally lead to a final column eccentricity smaller than the nominal value based on gross moments of inertia. Final eccentricities have been noted in this study as low as 0.52 times the nominal and as large as 2.12 times the nominal. A majority of cases show an increase too large to neglect in design.

These variations occur even for short columns. To eliminate column deflection effects, this study utilized columns with $h/t=1$. The analysis deals with members as line elements between joints of point size.

Scope of analysis

With significant variables* including r', p, and p' of beam, p_t of column, f_c', f_y, and nominal e/t of

*Symbols here are those of ACI 318-63.

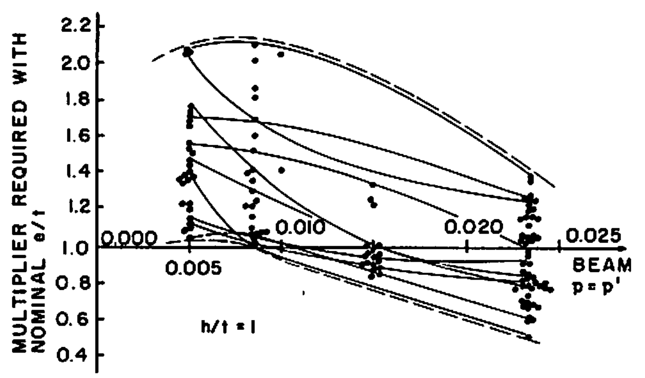


Fig. 1 — Computer solution values for all practical single cases investigated; dashed boundaries are for reference to Fig. 3

ACI member **Hajime Okamura** is an associate professor of civil engineering at the University of Tokyo, Tokyo, Japan. He received his BS in 1961, MS in 1963, and PhD in 1966 from the University of Tokyo. From 1966 to 1968 he was at The University of Texas at Austin on post-doctoral work. In 1968 he received the Yoshida Prize from the Japan Society of Civil Engineers for one of his papers.

ACI member **Shriniwas N. Pagay** is assistant professor of applied science, Portland State University, Portland, Ore. After obtaining his master's in power engineering in 1959 from the Indian Institute of Science, he received an MS degree in civil engineering from State University of Iowa (1962) and a PhD degree in structures from the University of Texas (1970). Dr. Pagay's research work includes reinforced concrete structures and materials, computer applications, and lightweight concrete.

Long-time ACI member **Phil M. Ferguson** is professor of civil engineering, The University of Texas at Austin. Currently, he is a member of ACI Committee 408, Bond Stress, and a member of ACI Committee 318, Standard Building Code. A frequent contributor to the Journal, Professor Ferguson has been active in teaching and research for over three decades. He is a past-president of the Institute and has served on its Board of Direction. He was one of eight delegates who represented ACI in Japan at the recent U.S.-Japan exchange of views on research in concrete. He authored the widely-known textbook, "Reinforced Concrete Fundamentals."

the column, the possible combination of variables is almost endless, even after excluding obviously impractical combinations. This report covers 149 computer analyses of frames containing columns in single curvature, of which 28 cases are found to be impractical because of premature beam hinging.[1] This computer program has given results checking closely with all test frame data to date.

Because this study was directed ultimately toward long columns, the computer cases emphasize single curvature braced columns carrying equal end moments. Braced long columns in double curvature are discussed briefly in the full report.

Assumptions and limitations

The analyses are based largely on interior columns such as occur in a uniform frame under a checkerboard loading, with dead load omitted. Columns are assumed the same in all stories involved and beams are rectangular, uniform in size and span. All unloaded members are in an exact single curvature pattern and all loaded beams develop equal and opposite end rotation angles.

The assumed beams have full length steel, equal in tension and in compression, of 60 ksi (4200 kg/cm^2) yield strength, with $0.15t$ cover over center of steel, where t is the over-all beam depth. Concrete strength for beams and columns is the same. Tension on concrete is neglected and the beam is assumed cracked throughout.

The columns are rectangular in shape, with bars on the two faces most effective in resisting moment at a cover to center of bar usually $0.15t$, corresponding to the familiar gt value of $0.70t$. Yield strength of column steel is 40 to 80 ksi (4200 to 8400 kg/cm^2) with 3 to 5 ksi (210 to 350 kg/cm^2) concrete. A few columns with gt of $0.85t$ and 80 ksi (5600 kg/cm^2) steel were included. Neither the time effect nor lightweight concrete is considered. The load-moment curve for each given column was developed under progressive loading and compared with the column interaction curve. The resulting data are recorded in tables of $P_{short}/P_{nominal}$ and (final e/t)/(nominal e/t) which cannot be reproduced in a summary report.

EFFECTS OF CRACKING AND INELASTIC ACTION, SINGLE CURVATURE, $h/t = 1$

General behavior patterns

Data were studied against individual variables, as for beam steel in Fig. 1, after eliminating impractical combinations not suitable for h/t as large as 3. With the largest p the eccentricity ratios range between 0.52 and 1.36, while with the very small p all are greater than unity. The scatter is less at $p = p' = 0.0150$ only because a narrowed range of variables was used in this case. It is obvious that beam reinforcement is not the only important variable.

For $r' = 1$,* the different behavior of a column (with minimum steel) under increasing load when light and heavy beam steel is used is shown in Fig. 2a. The radial line for nominal e/t indicates how ordinary analysis assumes the short column load and moment to increase linearly to failure at the interaction boundary, at a load of 349 kips† when a nominal 10 in† square column is used. The lightly reinforced beam loses more of its stiffness when cracked and initially shifts much more moment to the column. The column then fails at $P = 309$ kips, soon after its heavier stressed layer of steel yields; about a third of the original increase in e/t is recovered as the concrete reaches the flatter portion of its stress-strain curve. The beam with heavy steel loses less stiffness when cracked and shifts the initial column e/t less from the nominal value, but it permits even a larger recovery. The drift back reflects primarily the inelastic response of the concrete, exaggerated where the column steel yields (at about $P = 300$ kips). The varying stiffness of column and beam throughout the loading finally determines the ultimate moment distribution at the joint and the real ultimate e/t in the short column.

The effects of varying the column reinforcement p_t, f_c', and f_y are shown separately in Fig. 2(b), 2(c), and 2(d). Detailed comparisons of other variables must be omitted here. In general, large r' values lead to load ratios closer to unity and r' of 0.5 is particularly divergent and difficult to estimate. A true and general mathematical expression for either P_s/P_{nom} or $(e/t)_s/(e/t)_{nom}$ would obviously be quite complex.

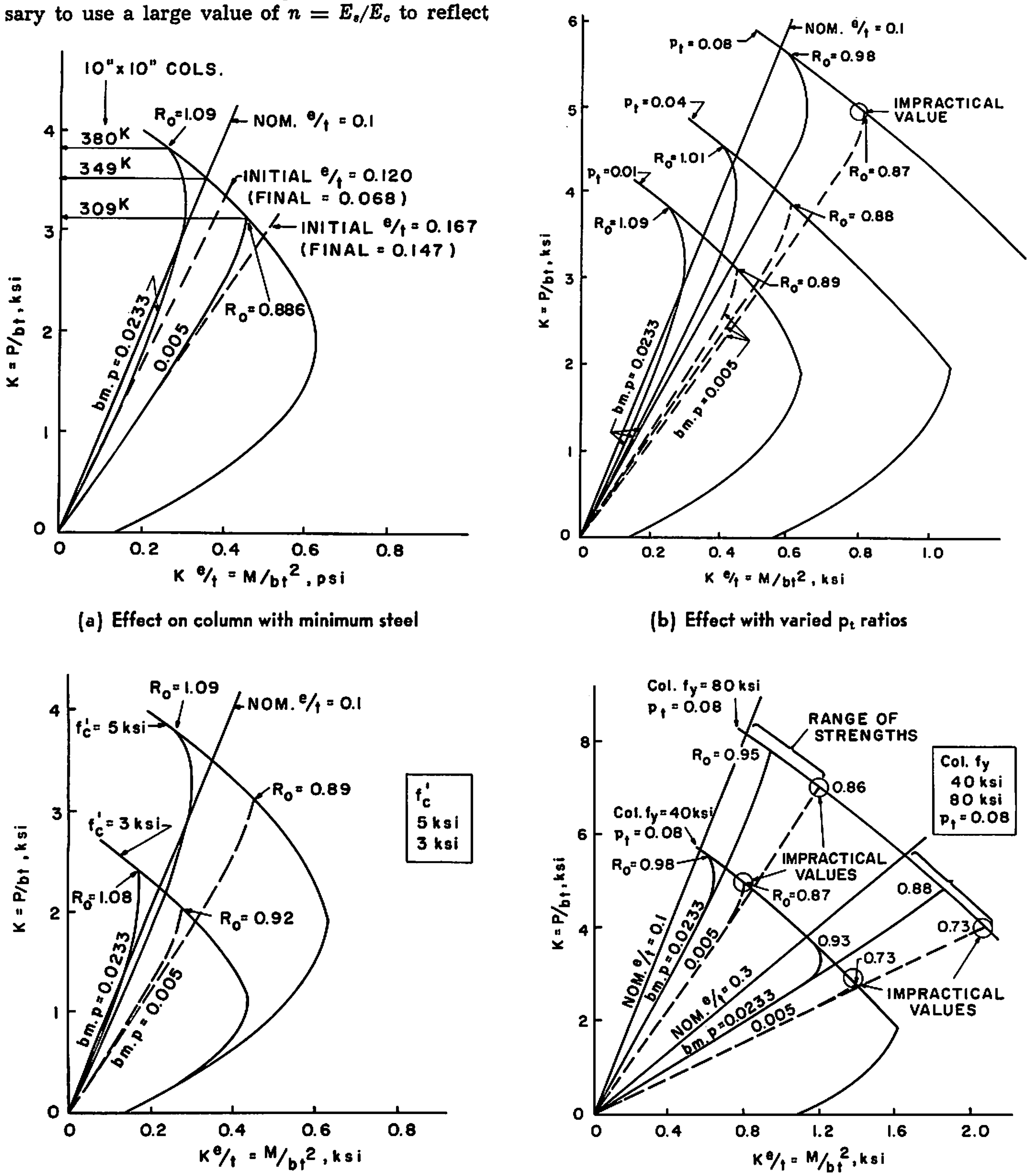
*All r' values used in this report are nominal values, that is, values computed on the basis of an elastic frame analysis assuming moments of inertia of the gross concrete and neglecting the effect of reinforcement.
†1 kip = 454 kg, 10 in. = 25.4 cm.

SEMIRATIONAL ANALYSIS

At early loading stages, for all practical purposes the cracked beam and the short uncracked column act elastically to establish the initial e/t.

With the concrete at a high stress, it is necessary to use a large value of $n = E_s/E_c$ to reflect the inelastic concrete behavior in an elastic frame analysis; the n for the compression steel then drops sharply if that steel yields.

A single procedure to represent both elastic and inelastic concepts can be semirational. By trial it



(a) Effect on column with minimum steel

(b) Effect with varied p_t ratios

(c) Effect with varied concrete strength $f_{c'}$ (column and beam)

(d) Effect of yield strength f_y of column steel

Fig. 2 — Effect of beam steel on various short columns in single curvature; $h/t = 1$, $r' = 1$, $g = 0.7$, nominal $e/t = 0.1$; except as noted, $f_c' = 5$ ksi, column $p_t = 0.01$, column $f_y = 40$ ksi; $R_o = P_{short}/P_{nom}$ (multiply ksi by 70.3 to obtain kg/cm²)

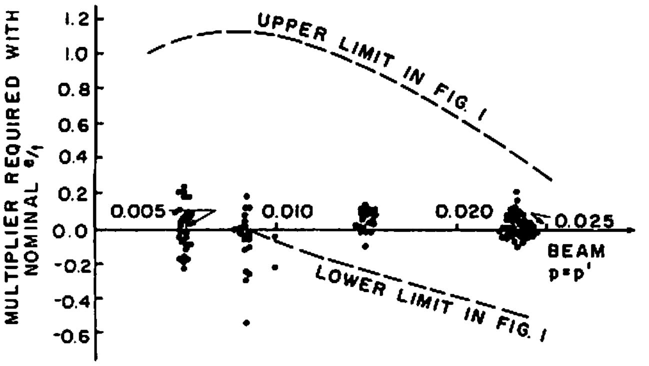


Fig. 3 — Error in empirical multipliers with $r' = 1$; negative value means empirical value is too small; limit lines from Fig. 1 indicate degree of improvement

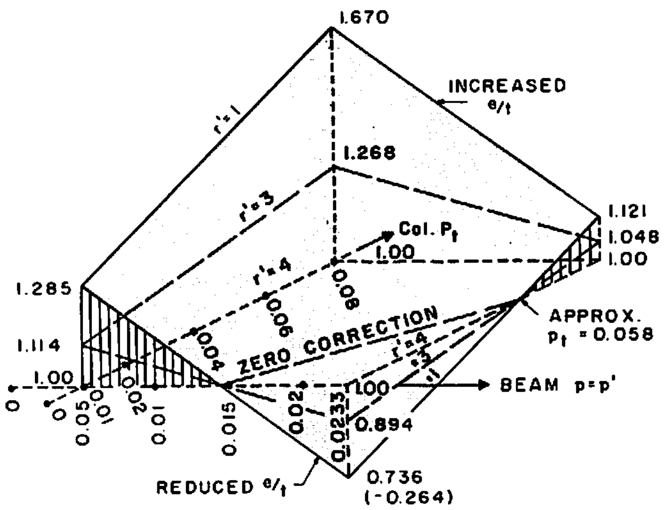


Fig. 4 — Graphical presentation of empirical relationships for e/t multiplier, single curvature cases

was found that the best elastic-type analysis resulted from using four times the usual n with the cracked section of the beam in combination with 2.5 times the usual n with an uncracked column section. Final e/t values calculated on this basis average 1.013 times the computer value, with a weighted error of 0.068, the weighted error being calculated as follows. The absolute error in e/t (based on the computer e/t as accurate) was first weighted by dividing by the nominal e/t for that case. These relative errors were squared, the squares totaled and averaged, and the weighted error is the square root of this average. A perfect solution would give the average ratio as 1.00 and the weighted error as zero, as both tend to converge somewhat together.

EMPIRICAL MULTIPLIER FOR ECCENTRICITY

An empirical multiplier for the nominal e/t in the single curvature case with $r' = 1$ can be expressed as:

$$1.38 + 5.5p_t - 30p$$

or

$$1.38 + 5.5\,(p_t - 5.5p)$$

where p_t is the ratio of column steel to the gross concrete area of the column and p is A_s/bd for the beam (based on $d = 0.85t$ and $p' = p$). The absolute errors in these multipliers, compared to the computer analyses, are plotted for $r' = 1$ in Fig. 3 for comparison with Fig. 1. This multiplier equation, and others for other r' values, are shown graphically in Fig. 4.

CONCLUSIONS

Ordinary frame analysis based on gross concrete sections generally leads to column ultimate moments and eccentricities considerably in error, generally on the unsafe side, because of (1) the lower stiffness of the cracked beam, (2) the loss of stiffness of the concrete in the column near ultimate, and (3) sometimes the yielding of column compression steel on one face.

Short columns in single curvature in braced frames, over a considerable range of variables, have been analyzed by a computer procedure, assuming all beams with equal top and bottom steel. The eccentricities at ultimate ranged from 0.52 to 2.08 times the nominal values obtained from ordinary moment distribution. A smaller sample of short columns with double curvature was also studied in the full report.

Moment distribution procedures, based on radical changes in calculated member stiffness, are indicated as feasible. For many cases empirical relations designed to be used as multipliers applied to the nominal e/t (calculated from stiffness based on gross concrete areas) are also presented.

The procedures discussed here for short columns provide a sound base for a study of the effect of length on column strength, since several inelastic member effects have here been identified as separate from the effect of length on column strength.

ACKNOWLEDGMENT

This paper summarizes a part of a study financed by the Reinforced Concrete Research Council and by Naval Facilities Engineering Command in cooperation with other government agencies.

REFERENCE

1. Pagay, Shriniwas N.; Ferguson, Phil M.; and Breen, John, E., "Importance of Beam Properties on Concrete Column Behavior," ACI JOURNAL, *Proceedings* V. 67, No. 10, Oct. 1970, pp. 808-815.

This paper was received by the Institute Dec. 24, 1969.

NOTE: This is an ACI digest paper. The full paper will not be published in the JOURNAL but xerographic, or similar, copies are available from the American Concrete Institute headquarters, where it will be kept permanently on file, at a charge equal to cost of reproduction plus handling at time of request.